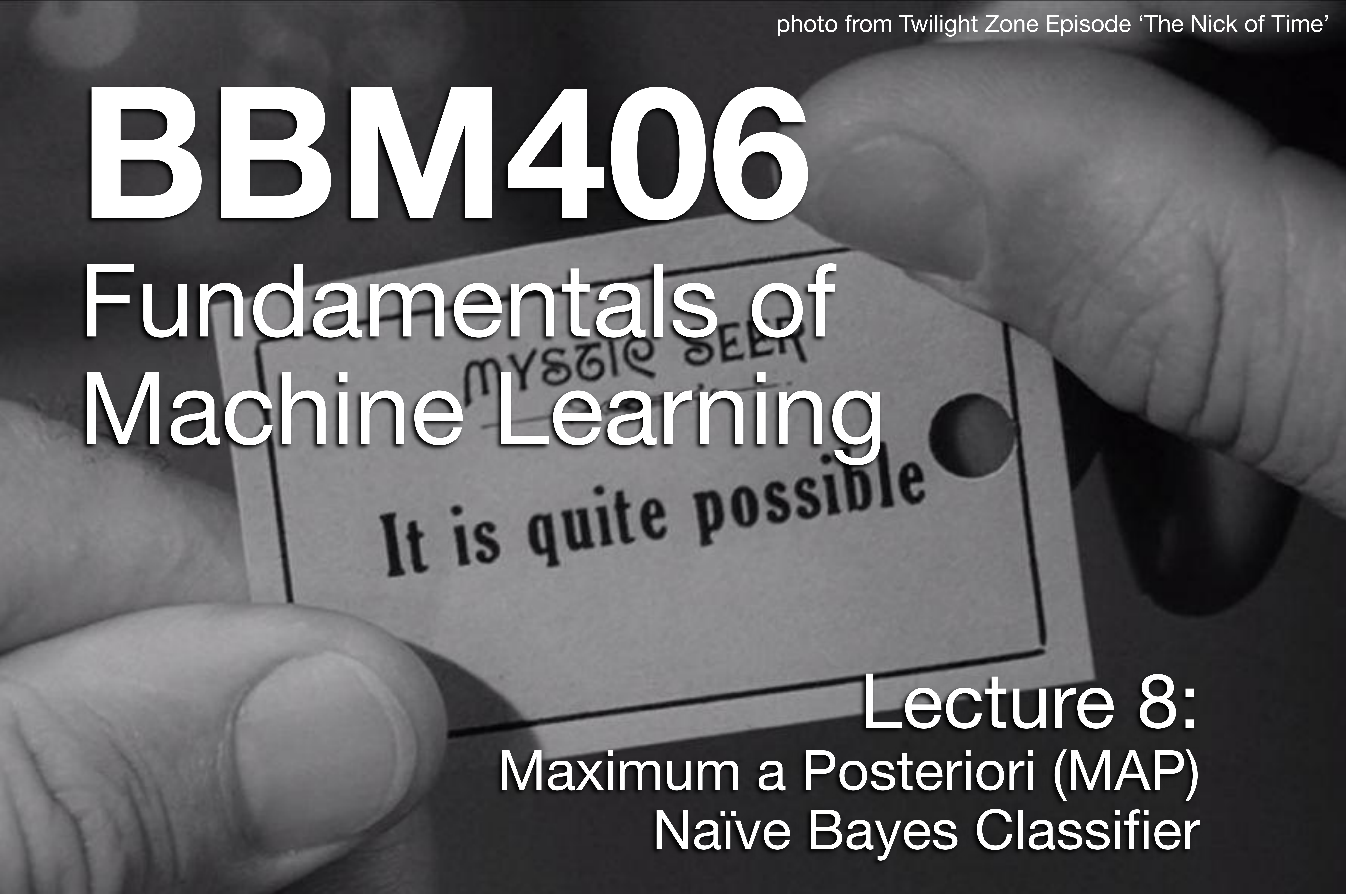
photo from Twilight Zone Episode ‘The Nick of Time’



BBM406

Fundamentals of Machine Learning

Lecture 8:

Maximum a Posteriori (MAP)
Naïve Bayes Classifier

HACETTEPE UNIVERSITY COMPUTER VISION LAB

Aykut Erdem // Hacettepe University // Fall 2019

Recap: MLE

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

slide by Barnabás Póczos & Aarti Singh

Today

- Maximum a Posteriori (MAP)

- Bayes rule
  - Naïve Bayes Classifier

- Application
  - Text classification
  - “Mind reading” = fMRI data processing

What about prior knowledge?
(MAP Estimation)

What about prior knowledge?

We know the coin is “close” to 50-50. What can we do now?

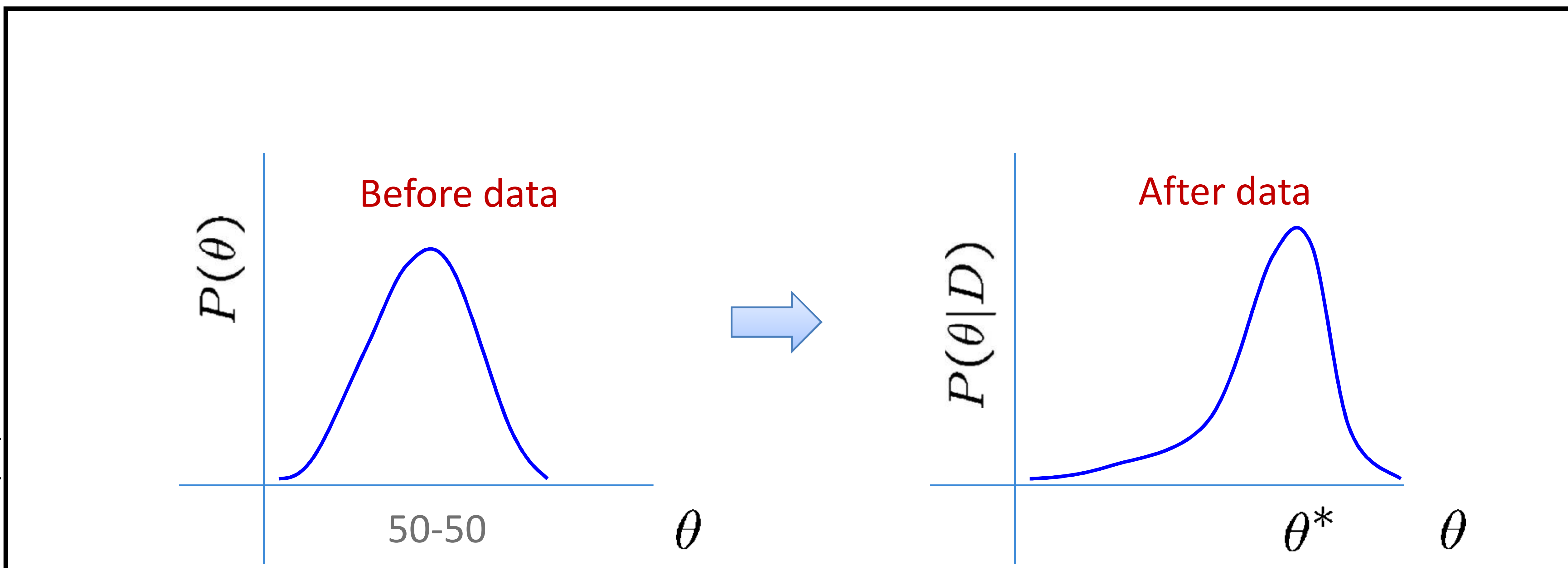
The Bayesian way…

What about prior knowledge?

We know the coin is "close" to 50-50. What can we do now?

The Bayesian way…

Rather than estimating a single θ, we obtain a distribution over possible values of θ

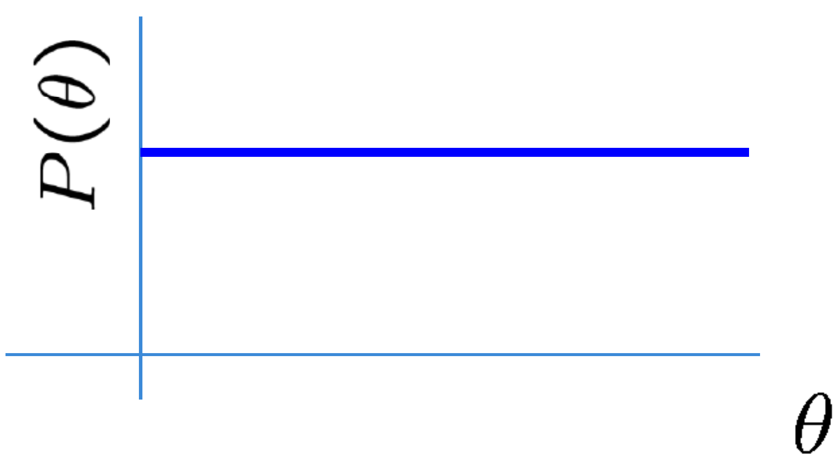


slide by Barnabás Póczos & Aarti Singh

Prior distribution

- What prior? What distribution do we want for a prior?
  - Represents expert knowledge (philosophical approach)
  - Simple posterior form (engineer's approach)

- Uninformative priors:
  - Uniform distribution



- Conjugate priors:
  - Closed-form representation of posterior
  - P(θ) and P(θ|D) have the same form

slide by Barnabás Póczos & Aarti Singh

In order to proceed we will need:

Bayes Rule

Bayes, Thomas (1763) An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London,* **53:370-418**

slide by Barnabás Póczos & Aarti Singh

Chain Rule & Bayes Rule

Chain rule:

$$P(X,Y) = P(X|Y)P(Y) = P(Y|X)P(X)$$

Bayes rule:

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)}$$

Bayes rule is important for reverse conditioning.

slide by Barnabás Póczos & Aarti Singh

Bayesian Learning

- Use Bayes rule:

$$P(\theta \mid \mathcal{D}) = \frac{P(\mathcal{D} \mid \theta) P(\theta)}{P(\mathcal{D})}$$

- Or equivalently:

$$\underbrace{P(\theta \mid \mathcal{D})}_{\text{posterior}} \propto \underbrace{P(\mathcal{D} \mid \theta)}_{\text{likelihood}} \underbrace{P(\theta)}_{\text{prior}}$$

Bayes, Thomas (1763) An essay towards solving a problem in the doctrine of chances. *Philosophical Transactions of the Royal Society of London*, **53:370-418**

slide by Barnabás Póczos & Aarti Singh

MAP estimation for Binomial distribution

Coin flip problem

Likelihood is Binomial $P(\mathcal{D} \mid \theta) = \binom{n}{\alpha_H} \theta^{\alpha_H} (1-\theta)^{\alpha_T}$

If the prior is Beta distribution,

$$P(\theta) = \frac{\theta^{\beta_H - 1}(1-\theta)^{\beta_T - 1}}{B(\beta_H, \beta_T)} \sim Beta(\beta_H, \beta_T)$$

$\Rightarrow$ posterior is Beta distribution

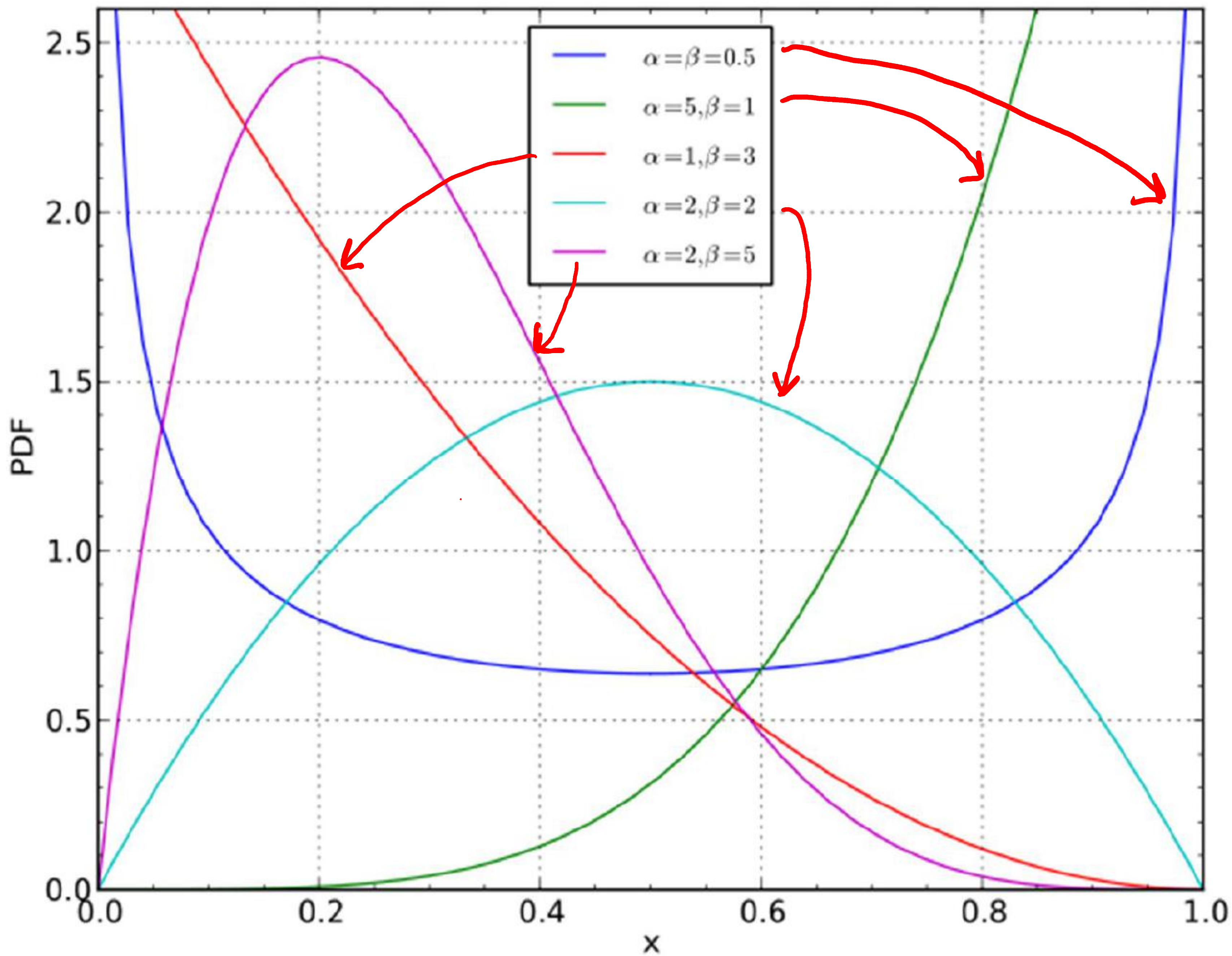
$$P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$$

P(θ) and P(θ| D) have the same form! [Conjugate prior]

$$\widehat{\theta}_{MAP} = \arg\max_{\theta} P(\theta \mid D) = \arg\max_{\theta} P(D \mid \theta) P(\theta) = \frac{\alpha_H + \beta_H - 1}{\alpha_H + \beta_H + \alpha_T + \beta_T - 2}$$

slide by Barnabás Póczos & Aarti Singh

Beta distribution

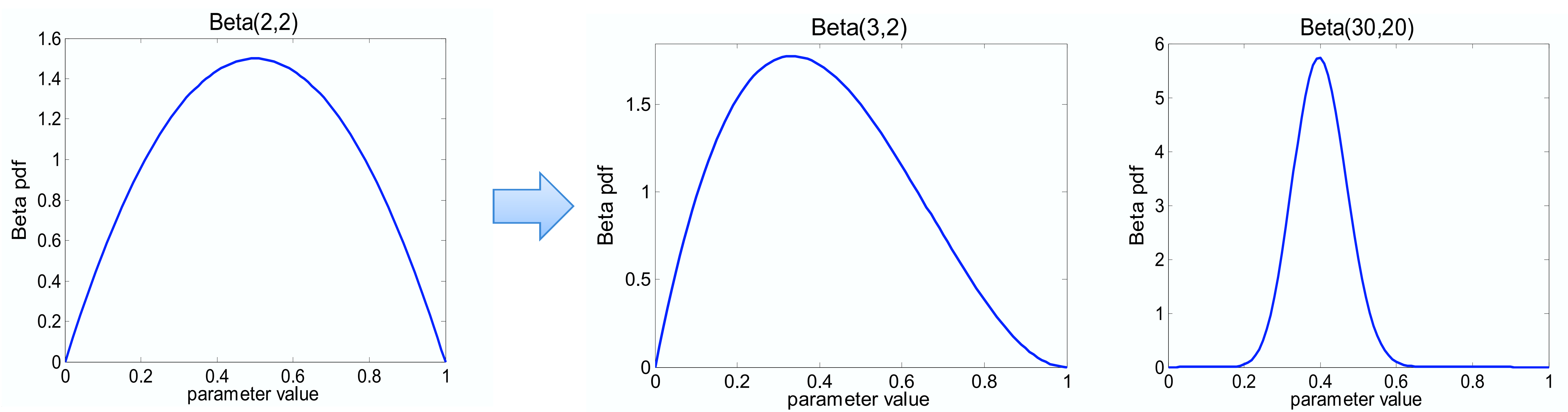


More concentrated as values of α, β increase

slide by Barnabás Póczos & Aarti Singh

Beta conjugate prior

$$P(\theta) \sim Beta(\beta_H, \beta_T) \qquad P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$$



As $n = \alpha_H + \alpha_T$ increases

As we get more samples, effect of prior is “washed out”

slide by Barnabás Póczos & Aarti Singh

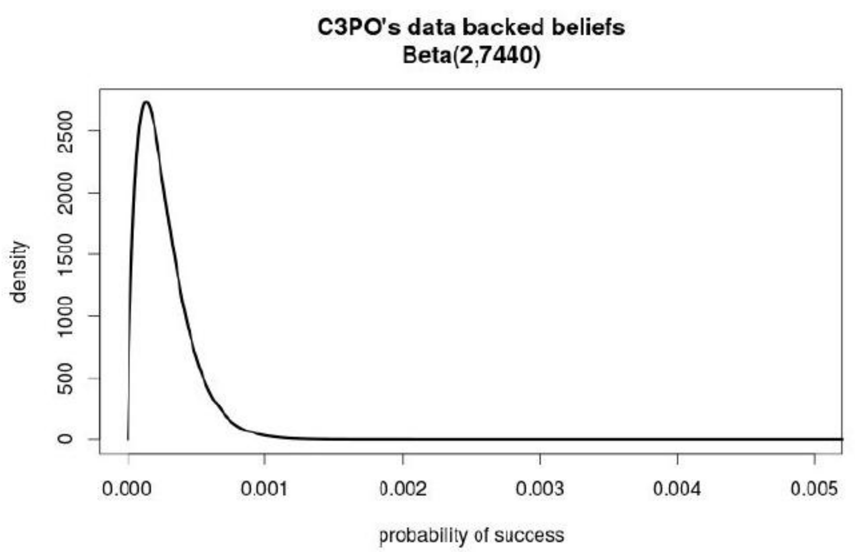
Han Solo and Bayesian Priors

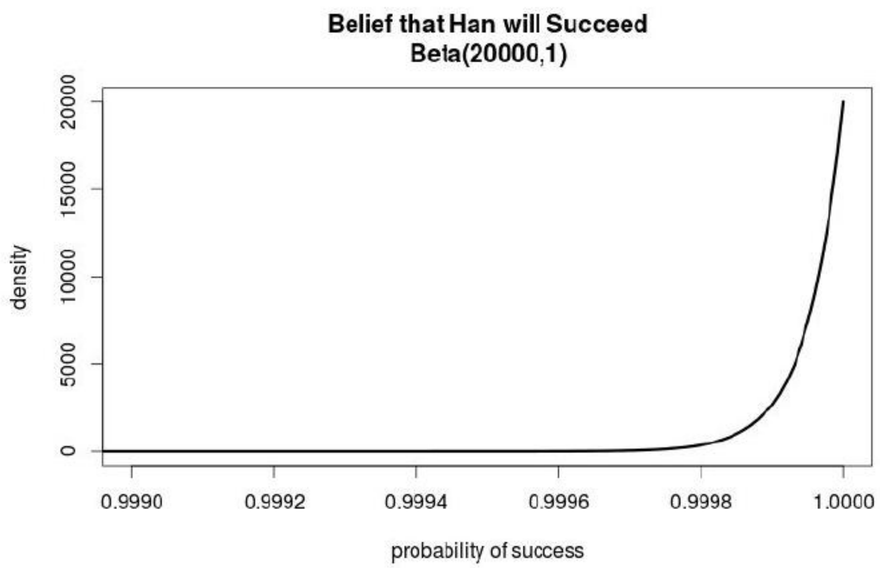
C3PO: Sir, the possibility of successfully navigating an asteroid field is approximately 3,720 to 1!

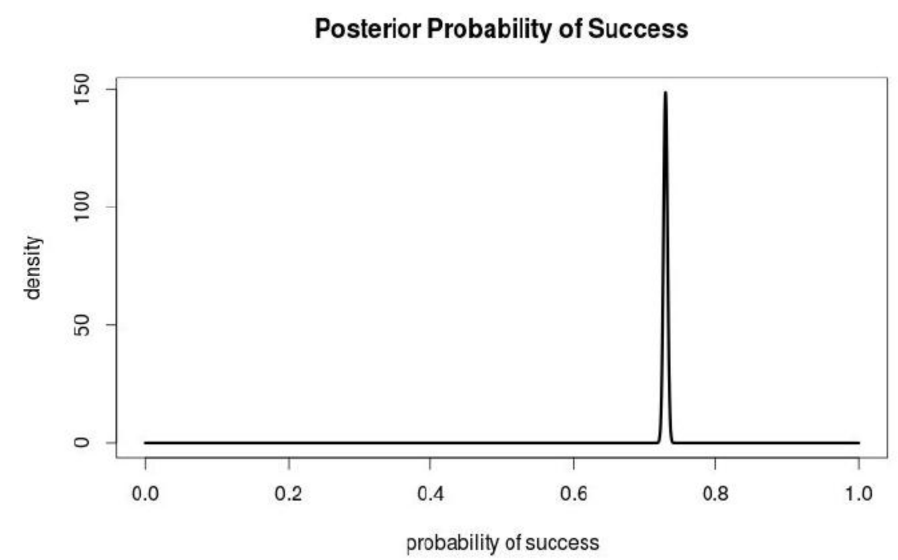
Han: Never tell me the odds!

$$P(\theta) \sim Beta(\beta_H, \beta_T)$$

$$P(\theta|D) \sim Beta(\beta_H + \alpha_H, \beta_T + \alpha_T)$$







https://www.countbayesie.com/blog/2015/2/18/hans-solo-and-bayesian-priors

MLE vs. MAP

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

MLE vs. MAP

- Maximum Likelihood estimation (MLE)

  Choose value that maximizes the probability of observed data

$$\widehat{\theta}_{MLE} = \arg\max_{\theta} P(D|\theta)$$

- Maximum *a posteriori* (MAP) estimation

  Choose value that is most probable given observed data and prior belief

$$\widehat{\theta}_{MAP} = \arg\max_{\theta} P(\theta|D)$$
$$= \arg\max_{\theta} P(D|\theta)P(\theta)$$

When is MAP same as MLE?

From Binomial to Multinomial

Example: Dice roll problem (6 outcomes instead of 2)

Likelihood is ~ Multinomial($\theta = \{\theta_1, \theta_2, \ldots, \theta_k\}$)

$$P(\mathcal{D} \mid \theta) = \theta_1^{\alpha_1} \theta_2^{\alpha_2} \ldots \theta_k^{\alpha_k}$$

If prior is Dirichlet distribution,

$$P(\theta) = \frac{\prod_{i=1}^{k} \theta_i^{\beta_i - 1}}{B(\beta_1, \ldots, \beta_k)} \sim \text{Dirichlet}(\beta_1, \ldots, \beta_k)$$

Then posterior is Dirichlet distribution

$$P(\theta|D) \sim \text{Dirichlet}(\beta_1 + \alpha_1, \ldots, \beta_k + \alpha_k)$$

For Multinomial, conjugate prior is Dirichlet distribution.

http://en.wikipedia.org/wiki/Dirichlet_distribution

slide by Barnabás Póczos & Aarti Singh

Bayesians vs. Frequentists

You are no good when sample is small

You give a different answer for different priors

slide by Barnabás Póczos & Aarti Singh

Application of Bayes Rule

slide by Barnabás Póczos & Aarti Singh

AIDS test (Bayes rule)

Data

- **Approximately 0.1% are infected**
- **Test detects all infections**
- **Test reports positive for 1% healthy people**

Probability of having AIDS if test is positive

$$P(a=1|t=1) = \frac{P(t=1|a=1)P(a=1)}{P(t=1)}$$

$$= \frac{P(t=1|a=1)P(a=1)}{P(t=1|a=1)P(a=1) + P(t=1|a=0)P(a=0)}$$

$$= \frac{1 \cdot 0.001}{1 \cdot 0.001 + 0.01 \cdot 0.999} = 0.091$$

Only 9%!...

slide by Barnabás Póczos & Aarti Singh

Improving the diagnosis

Use a weaker follow-up test!

- Approximately 0.1% are infected
- Test 2 reports positive for 90% infections
- Test 2 reports positive for 5% healthy people

$$P(a=0|t_1=1,t_2=1) = \frac{P(t_1=1,t_2=1|a=0)P(a=0)}{P(t_1=1,t_2=1|a=1)P(a=1)+P(t_1=1,t_2=1|a=0)P(a=0)}$$

$$= \frac{0.01\cdot 0.05\cdot 0.999}{1\cdot 0.9\cdot 0.001 + 0.01\cdot 0.05\cdot 0.999} = 0.357$$

$$P(a=1|t_1=1,t_2=1) = 0.643$$

64%!...

AIDS test (Bayes rule)

Why can't we use Test 1 twice?

- Outcomes are **not** independent,
- but tests 1 and 2 **conditionally independent (by assumption)**:

$$p(t_1, t_2|a) = p(t_1|a) \cdot p(t_2|a)$$

slide by Barnabás Póczos & Aarti Singh

The Naïve Bayes Classifier

slide by Barnabás Póczos & Aarti Singh

Data for spam filtering

- date
- time
- recipient path
- IP number
- sender
- encoding
- many more features

```
Delivered-To: alex.smola@gmail.com
Received: by 10.216.47.73 with SMTP id s51cs361171web;
        Tue, 3 Jan 2012 14:17:53 -0800 (PST)
Received: by 10.213.17.145 with SMTP id s17mr2519891eba.147.1325629071725;
        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
Return-Path: <alex+caf_=alex.smola=gmail.com@smola.org>
Received: from mail-ey0-f175.google.com (mail-ey0-f175.google.com [209.85.215.175])
        by mx.google.com with ESMTPS id n4si29264232eef.57.2012.01.03.14.17.51
        (version=TLSv1/SSLv3 cipher=OTHER);
        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
Received-SPF: neutral (google.com: 209.85.215.175 is neither permitted nor denied by best
guess record for domain of alex+caf_=alex.smola=gmail.com@smola.org) client-
ip=209.85.215.175;
Authentication-Results: mx.google.com; spf=neutral (google.com: 209.85.215.175 is neither
permitted nor denied by best guess record for domain of
alex+caf_=alex.smola=gmail.com@smola.org)
smtp.mail=alex+caf_=alex.smola=gmail.com@smola.org; dkim=pass (test mode)
header.i=@googlemail.com
Received: by eaal1 with SMTP id l1so15092746eaa.6
        for <alex.smola@gmail.com>; Tue, 03 Jan 2012 14:17:51 -0800 (PST)
Received: by 10.205.135.18 with SMTP id ie18mr5325064bkc.72.1325629071362;
        Tue, 03 Jan 2012 14:17:51 -0800 (PST)
X-Forwarded-To: alex.smola@gmail.com
X-Forwarded-For: alex@smola.org alex.smola@gmail.com
Delivered-To: alex@smola.org
Received: by 10.204.65.198 with SMTP id k6cs206093bki;
        Tue, 3 Jan 2012 14:17:50 -0800 (PST)
Received: by 10.52.88.179 with SMTP id bh19mr10729402vdb.38.1325629068795;
        Tue, 03 Jan 2012 14:17:48 -0800 (PST)
Return-Path: <althoff.tim@googlemail.com>
Received: from mail-vx0-f179.google.com (mail-vx0-f179.google.com [209.85.220.179])
        by mx.google.com with ESMTPS id dt4si11767074vdb.93.2012.01.03.14.17.48
        (version=TLSv1/SSLv3 cipher=OTHER);
        Tue, 03 Jan 2012 14:17:48 -0800 (PST)
Received-SPF: pass (google.com: domain of althoff.tim@googlemail.com designates
209.85.220.179 as permitted sender) client-ip=209.85.220.179;
Received: by vcbf13 with SMTP id f13so11295098vcb.10
        for <alex@smola.org>; Tue, 03 Jan 2012 14:17:48 -0800 (PST)
DKIM-Signature: v=1; a=rsa-sha256; c=relaxed/relaxed;
        d=googlemail.com; s=gamma;
        h=mime-version:sender:date:x-google-sender-auth:message-id:subject
         :from:to:content-type;
        bh=WCbdZ5sXac25dpH02XcRyDOdts993hKwsAVXpGrFh0w=;
        b=WK2B2+ExWnf/gvTkw6uUvKuP4XeoKnlJq3USYTm0RARK8dSFjyOQsIHeAP9Yssxp6O
         7ngGoTzYqd+ZsyJfvQcLAWp1PCJhG8AMcnqWkx0NMeoFvIp2HQooZwxSOCx5ZRgY+7qX
         uIbbdna4lUDXj6UFe16SpLDCkptd8OZ3gr7+o=
MIME-Version: 1.0
Received: by 10.220.108.81 with SMTP id e17mr24104004vcp.67.1325629067787;
 Tue, 03 Jan 2012 14:17:47 -0800 (PST)
Sender: althoff.tim@googlemail.com
Received: by 10.220.17.129 with HTTP; Tue, 3 Jan 2012 14:17:47 -0800 (PST)
Date: Tue, 3 Jan 2012 14:17:47 -0800
X-Google-Sender-Auth: 6bwi6D17HjZIkxOEol38NZzyeHs
Message-ID: <CAFJJHDGPBW+SdZg0MdAABiAKydDk9tpeMoDijYGjoGO-WC7osg@mail.gmail.com>
Subject: CS 281B. Advanced Topics in Learning and Decision Making
```

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Assumption

Naïve Bayes assumption: Features X_1 and X_2 are conditionally independent given the class label Y:

$$P(X_1, X_2|Y) = P(X_1|Y)P(X_2|Y)$$

More generally:

$$P(X_1 \ldots X_d|Y) = \prod_{i=1}^{d} P(X_i|Y)$$

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Assumption, Example

Task: Predict whether or not a picnic spot is enjoyable

Training Data: $X = (X_1 \quad X_2 \quad X_3 \quad \ldots \quad \ldots \quad X_d) \quad Y$

n rows

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Assumption, Example

Task: Predict whether or not a picnic spot is enjoyable

Training Data: $X = (X_1 \quad X_2 \quad X_3 \quad \ldots \quad \ldots \quad X_d) \quad Y$

n rows

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Naïve Bayes assumption:

$$P(X_1 \ldots X_d | Y) = \prod_{i=1}^{d} P(X_i | Y)$$

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Assumption, Example

Task: Predict whether or not a picnic spot is enjoyable

Training Data: $X = (X_1 \quad X_2 \quad X_3 \quad \dots \quad \dots \quad X_d) \quad Y$

n rows

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Naïve Bayes assumption:

$$P(X_1 \dots X_d | Y) = \prod_{i=1}^{d} P(X_i | Y)$$

How many parameters to estimate?

(X is composed of d binary features,

Y has K possible class labels)

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Assumption, Example

Task: Predict whether or not a picnic spot is enjoyable

Training Data: $X = (X_1 \quad X_2 \quad X_3 \quad \ldots \quad \ldots \quad X_d) \quad Y$

n rows

Sky	Temp	Humid	Wind	Water	Forecst	EnjoySpt
Sunny	Warm	Normal	Strong	Warm	Same	Yes
Sunny	Warm	High	Strong	Warm	Same	Yes
Rainy	Cold	High	Strong	Warm	Change	No
Sunny	Warm	High	Strong	Cool	Change	Yes

Naïve Bayes assumption:

$$P(X_1 \ldots X_d | Y) = \prod_{i=1}^{d} P(X_i | Y)$$

How many parameters to estimate?

(X is composed of d binary features,
Y has K possible class labels)

$(2^d-1)K$ vs $(2-1)dK$

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Classifier

Given:

- Class prior $P(Y)$
- d conditionally independent features $X_1, \ldots X_d$ given the class label Y
- For each X_i feature, we have the conditional likelihood $P(X_i|Y)$

Naïve Bayes Decision rule:

$$\begin{aligned} f_{NB}(\mathbf{x}) &= \arg\max_y \; P(x_1, \ldots, x_d \mid y) P(y) \\ &= \arg\max_y \prod_{i=1}^{d} \; P(x_i|y) P(y) \end{aligned}$$

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Algorithm for discrete features

Training data: $\{(X^{(j)}, Y^{(j)})\}_{j=1}^{n}$

$$X^{(j)} = (X_1^{(j)}, \ldots, X_d^{(j)})$$

n *d*-dimensional discrete features + K class labels

$$f_{NB}(\mathbf{x}) = \arg\max_y \prod_{i=1}^{d} P(x_i|y)P(y)$$

We need to estimate these probabilities!

Estimate them with MLE (Relative Frequencies)!

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Algorithm for discrete features

$$f_{NB}(\mathbf{x}) = \arg\max_y \prod_{i=1}^{d} P(x_i|y)P(y)$$

We need to estimate these probabilities!

Estimators

For Class Prior

$$\widehat{P}(y) = \frac{\{\#j : Y^{(j)} = y\}}{n}$$

For Likelihood

$$\frac{\widehat{P}(x_i, y)}{\widehat{P}(y)} = \frac{\{\#j : X_i^{(j)} = x_i, Y^{(j)} = y\}/n}{\{\#j : Y^{(j)} = y\}/n}$$

NB Prediction for test data: $X = (x_1, \ldots, x_d)$

$$Y = \arg\max_y \widehat{P}(y) \prod_{i=1}^{d} \frac{\widehat{P}(x_i, y)}{\widehat{P}(y)}$$

slide by Barnabás Póczos & Aarti Singh

Subtlety: Insufficient training data

What if you never see a training instance where $X_1 = a$ when $Y = b$?

For example,

there is no X_1='Earn' when Y='SpamEmail' in our dataset.

$$\Rightarrow P(X_1 = a, Y = b) = 0 \Rightarrow P(X_1 = a | Y = b) = 0$$

$$\boxed{\Rightarrow P(X_1 = a, X_2 ... X_n | Y) = P(X_1 = a | Y) \prod_{i=2}^{d} P(X_i | Y) = 0}$$

Thus, no matter what the values $X_2, \ldots, X_d$ take:

$$P(Y = b \mid X_1 = a, X_2, \ldots, X_d) = 0$$

What now???

slide by Barnabás Póczos & Aarti Singh

Naïve Bayes Alg — Discrete features

Training data: $\{(X^{(j)}, Y^{(j)})\}_{j=1}^{n}$ $\quad X^{(j)} = (X_1^{(j)}, \ldots, X_d^{(j)})$

Use your expert knowledge & apply prior distributions:

- Add m “virtual” examples
- Same as assuming conjugate priors

Assume priors: $Q(Y = b)$ $\quad Q(X_i = a, Y = b)$

MAP Estimate:

$$\widehat{P}(X_i = a | Y = b) = \frac{\{\#j : X_i^{(j)} = a, Y^{(j)} = b\} + mQ(X_i = a, Y = b)}{\{\#j : Y^{(j)} = b\} + \underbrace{mQ(Y = b)}_{\text{\# virtual examples with } Y = b}}$$

called Laplace smoothing

slide by Barnabás Póczos & Aarti Singh

Case Study:
Text Classification

Positive or negative movie review?

- unbelievably disappointing

- Full of zany characters and richly applied satire, and some great plot twists

- this is the greatest screwball comedy ever filmed

- It was pathetic. The worst part about it was the boxing scenes.

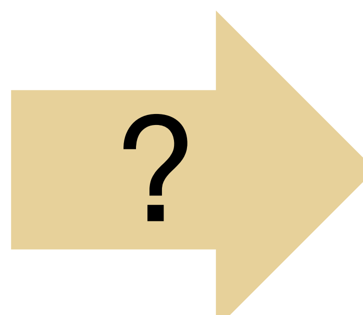
slide by Dan Jurafsky

What is the subject of this article?

MEDLINE Article


Brain and Cognition

Syntactic frame and verb bias in aphasia: Plausibility judgments of undergoer-subject sentences



MeSH Subject Category Hierarchy

- Antogonists and Inhibitors
- Blood Supply
- Chemistry
- Drug Therapy
- Embryology
- Epidemiology
- …

Text Classification

- Assigning subject categories, topics, or genres
- Spam detection
- Authorship identification
- Age/gender identification
- Language Identification
- Sentiment analysis
- …

slide by Dan Jurafsky

Text Classification: definition

- Input:
  - a document d
  - a fixed set of classes $C = \{c_1, c_2, \ldots, c_J\}$

- Output: a predicted class $c \in C$

Hand-coded rules

- Rules based on combinations of words or other features
  - spam: black-list-address OR (“dollars” AND“have been selected”)
- Accuracy can be high
  - If rules carefully refined by expert
- But building and maintaining these rules is expensive

slide by Dan Jurafsky

Text Classification and Naive Bayes

- Classify emails
  - Y = {Spam, NotSpam}

- Classify news articles
  - Y = {what is the topic of the article?}

What are the features X?

The text!

Let X_i represent i^{th} word in the document

slide by Barnabás Póczos & Aarti Singh

X_i represents i^{th} word in document


Article from rec.sport.hockey

Path: cantaloupe.srv.cs.cmu.edu!das-news.harvard.e
From: xxx@yyy.zzz.edu (John Doe)
Subject: Re: This year's biggest and worst (opinic
Date: 5 Apr 93 09:53:39 GMT

I can only comment on the Kings, but the most obvious candidate for pleasant surprise is Alex Zhitnik. He came highly touted as a defensive defenseman, but he's clearly much more than that. Great skater and hard shot (though wish he were more accurate). In fact, he pretty much allowed the Kings to trade away that huge defensive liability Paul Coffey. Kelly Hrudey is only the biggest disappointment if you thought he was any good to begin with. But, at best, he's only a mediocre goaltender. A better choice would be Tomas Sandstrom, though not through any fault of his own, but because some thugs in Toronto decided

slide by Barnabás Póczos & Aarti Singh

NB for Text Classification

A problem: The support of P(**X**|Y) is huge!

– Article at least 1000 words, $\mathbf{X}=\{X_1,\ldots,X_{1000}\}$

– X_i represents i^{th} word in document, i.e., the domain of X_i is the entire vocabulary, e.g., Webster Dictionary (or more).

$X_i \in \{1,\ldots,50000\} \Rightarrow K(1000^{50000}-1)$

parameters to estimate without the NB assumption….

$$h_{MAP}(\mathbf{x}) = \arg\max_{1\le k\le K} P(Y=k)P(X_1=x_1,\ldots,X_{1000}=x_{1000}|Y=k)$$

slide by Barnabás Póczos & Aarti Singh

NB for Text Classification

$X_i \in \{1,\ldots,50000\} \Rightarrow K(1000^{50000}-1)$ parameters to estimate....

NB assumption helps a lot!!!

If $P(X_i=x_i|Y=y)$ is the probability of observing word x_i at the i^{th} position in a document on topic y

$\Rightarrow$ 1000K(50000-1) parameters to estimate with NB assumption

NB assumption helps, but still lots of parameters to estimate.

$$h_{NB}(\mathbf{x}) = \arg\max_y P(y) \prod_{i=1}^{LengthDoc} P(X_i = x_i|y)$$

slide by Barnabás Póczos & Aarti Singh

Bag of words model

Typical additional assumption:

Position in document doesn't matter:

$P(X_i=x_i|Y=y) = P(X_k=x_i|Y=y)$

- "Bag of words" model – order of words on the page ignored
  The document is just a bag of words: i.i.d. words
- Sounds really silly, but often works very well!

$\Rightarrow$ K(50000-1) parameters to estimate

The probability of a document with words $x_1, x_2, \ldots$

$$\prod_{i=1}^{LengthDoc} P(x_i|y) = \prod_{w=1}^{W} P(w|y)^{count_w}$$

slide by Barnabás Póczos & Aarti Singh

The bag of words representation

$\gamma($

I love this movie! It's sweet, but with satirical humor. The dialogue is great and the adventure scenes are fun… It manages to be whimsical and romantic while laughing at the conventions of the fairy tale genre. I would recommend it to just about anyone. I've seen it several times, and I'm always happy to see it again whenever I have a friend who hasn't seen it yet.

$) = c$

slide by Dan Jurafsky

The bag of words representation

$\gamma($

I **love** this movie! It's **sweet**, but with **satirical** humor. The dialogue is **great** and the adventure scenes are **fun**… It manages to be **whimsical** and **romantic** while **laughing** at the conventions of the fairy tale genre. I would **recommend** it to just about anyone. I've seen it **several** times, and I'm always **happy** to see it **again** whenever I have a friend who hasn't seen it yet.

$) = c$

slide by Dan Jurafsky

The bag of words representation: using a subset of words

γ(

```
x love xxxxxxxxxxxxxxxx sweet
xxxxxxx satirical xxxxxxxxxx
xxxxxxxxxxx great xxxxxxx
xxxxxxxxxxxxxxxxxxx fun  xxxx
xxxxxxxxxxxxx whimsical xxxx
romantic xxxx  laughing
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxx recommend xxxxx
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
x several xxxxxxxxxxxxxxxxx
xxxxx  happy xxxxxxxxx again
xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx
xxxxxxxxxxxxxxxx
```

)=c

slide by Dan Jurafsky

The bag of words representation

$\gamma($

great	2
love	2
recommend	1
laugh	1
happy	1
...	...

$)=c$

slide by Dan Jurafsky

$$\hat{P}(c)=\frac{N_c}{N}$$

$$\hat{P}(w\mid c)=\frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

slide by Dan Jurafsky

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$$P(c)= \frac{3}{4}$$

$$P(j)= \frac{1}{4}$$

slide by Dan Jurafsky

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w \mid c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

Conditional Probabilities:

P(Chinese|c) = (5+1) / (8+6) = 6/14 = 3/7

P(Tokyo|c) = (0+1) / (8+6) = 1/14

P(Japan|c) = (0+1) / (8+6) = 1/14

P(Chinese|j) = (1+1) / (3+6) = 2/9

P(Tokyo|j) = (1+1) / (3+6) = 2/9

P(Japan|j) = (1+1) / (3+6) = 2/9

$$\hat{P}(c) = \frac{N_c}{N}$$

$$\hat{P}(w|c) = \frac{count(w,c)+1}{count(c)+|V|}$$

	Doc	Words	Class
Training	1	Chinese Beijing Chinese	c
	2	Chinese Chinese Shanghai	c
	3	Chinese Macao	c
	4	Tokyo Japan Chinese	j
Test	5	Chinese Chinese Chinese Tokyo Japan	?

Priors:

$$P(c) = \frac{3}{4}$$

$$P(j) = \frac{1}{4}$$

Conditional Probabilities:

$P(\text{Chinese}|c) = (5+1) / (8+6) = 6/14 = 3/7$

$P(\text{Tokyo}|c) = (0+1) / (8+6) = 1/14$

$P(\text{Japan}|c) = (0+1) / (8+6) = 1/14$

$P(\text{Chinese}|j) = (1+1) / (3+6) = 2/9$

$P(\text{Tokyo}|j) = (1+1) / (3+6) = 2/9$

$P(\text{Japan}|j) = (1+1) / (3+6) = 2/9$

Choosing a class:

$$P(c|d_5) \propto 3/4 * (3/7)^3 * 1/14 * 1/14 \approx 0.0003$$

$$P(j|d_5) \propto 1/4 * (2/9)^3 * 2/9 * 2/9 \approx 0.0001$$

slide by Dan Jurafsky

Twenty news groups results

Given 1000 training documents from each group
Learn to classify new documents according to which newsgroup it came from

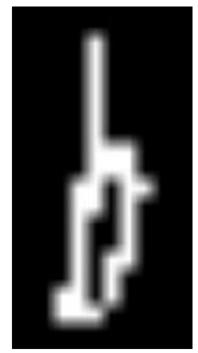
comp.graphics	misc.forsale
comp.os.ms-windows.misc	rec.autos
comp.sys.ibm.pc.hardware	rec.motorcycles
comp.sys.mac.hardware	rec.sport.baseball
comp.windows.x	rec.sport.hockey
alt.atheism	sci.space
soc.religion.christian	sci.crypt
talk.religion.misc	sci.electronics
talk.politics.mideast	sci.med
talk.politics.misc	
talk.politics.guns	

Naïve Bayes: 89% accuracy

slide by Barnabás Póczos & Aarti Singh

What if features are continuous?

e.g., character recognition: X_i is intensity at i^{th} pixel



Gaussian Naïve Bayes (GNB):

$$P(X_i = x \mid Y = y_k) = \frac{1}{\sigma_{ik}\sqrt{2\pi}} \; e^{\frac{-(x-\mu_{ik})^2}{2\sigma_{ik}^2}}$$

Different mean and variance for each class k and each pixel i.

Sometimes assume variance

- is independent of Y (i.e., σ_i),
- or independent of X_i (i.e., σ_k)
- or both (i.e., σ)

slide by Barnabás Póczos & Aarti Singh

Estimating parameters: Y discrete, X_i continuous

$$h_{NB}(\mathbf{x}) = \arg\max_y P(y) \prod_i P(X_i = x_i | y)$$

$$\approx \arg\max_k \widehat{P}(Y = k) \prod_i \mathcal{N}(\widehat{\mu}_{ik}, \widehat{\sigma}_{ik})$$

$$\boxed{\widehat{\mu}_{MLE} = \frac{1}{N}\sum_{j=1}^{N} x_j}$$

$$\boxed{\widehat{\sigma}^2_{unbiased} = \frac{1}{N-1}\sum_{j=1}^{N} (x_j - \widehat{\mu})^2}$$

slide by Barnabás Póczos & Aarti Singh

Estimating parameters: Y discrete, X_i continuous

Maximum likelihood estimates:

$$\boxed{\widehat{\mu}_{MLE} = \frac{1}{N}\sum_{j=1}^{N} x_j}$$

$$\widehat{\mu}_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k)} \sum_j X_i^j \delta(Y^j = y_k)$$

k^{th} class

j^{th} training image

i^{th} pixel in j^{th} training image

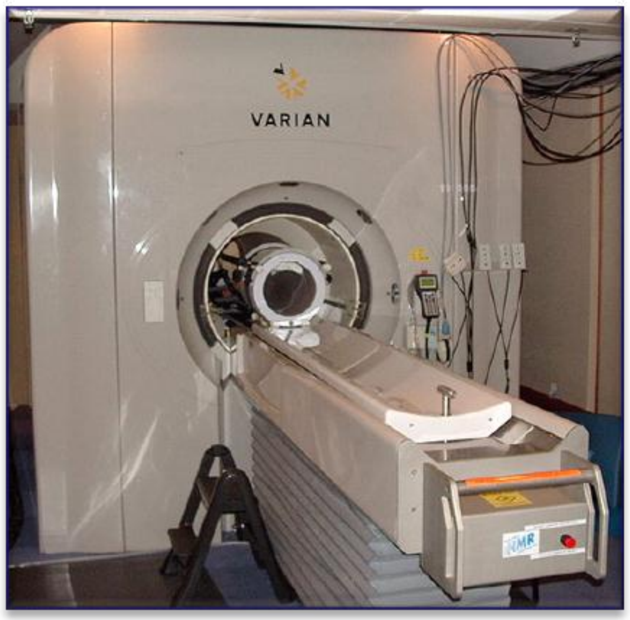
$$\boxed{\widehat{\sigma}^2_{unbiased} = \frac{1}{N-1}\sum_{j=1}^{N} (x_j - \widehat{\mu})^2}$$

$$\widehat{\sigma}^2_{ik} = \frac{1}{\sum_j \delta(Y^j = y_k) - 1} \sum_j (X_i^j - \widehat{\mu}_{ik})^2 \delta(Y^j = y_k)$$

slide by Barnabás Póczos & Aarti Singh

Case Study:
Classifying Mental States

Example: GNB for classifying mental states



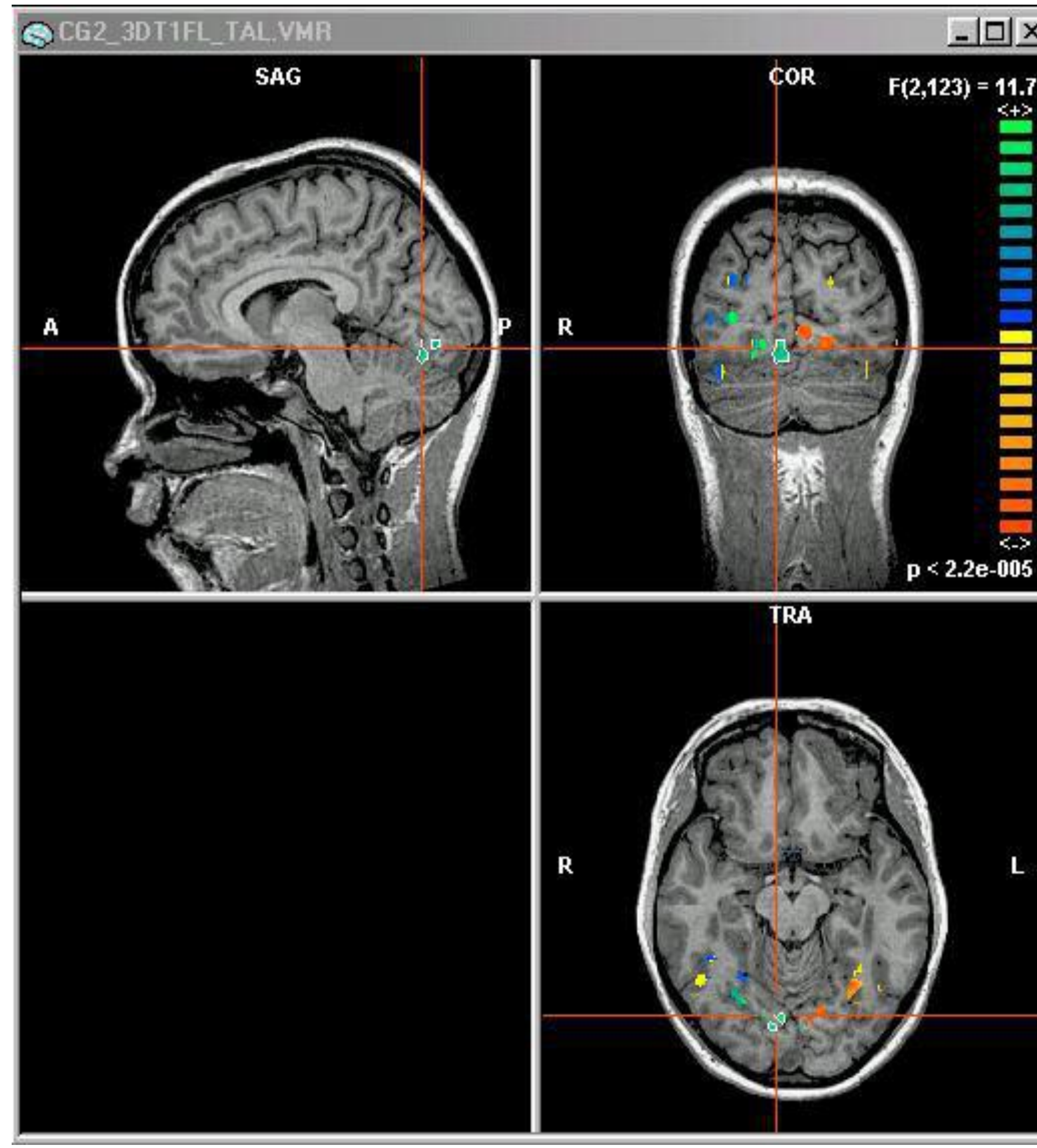
~1 mm resolution

~2 images per sec.

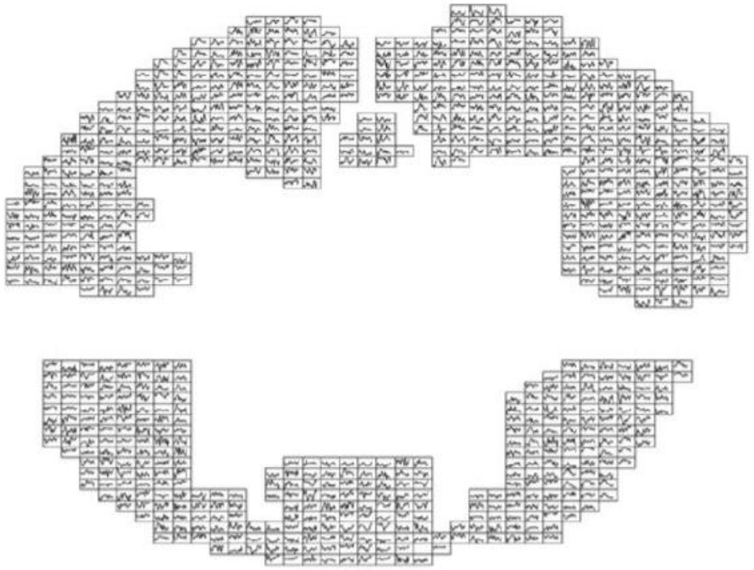
15,000 voxels/image

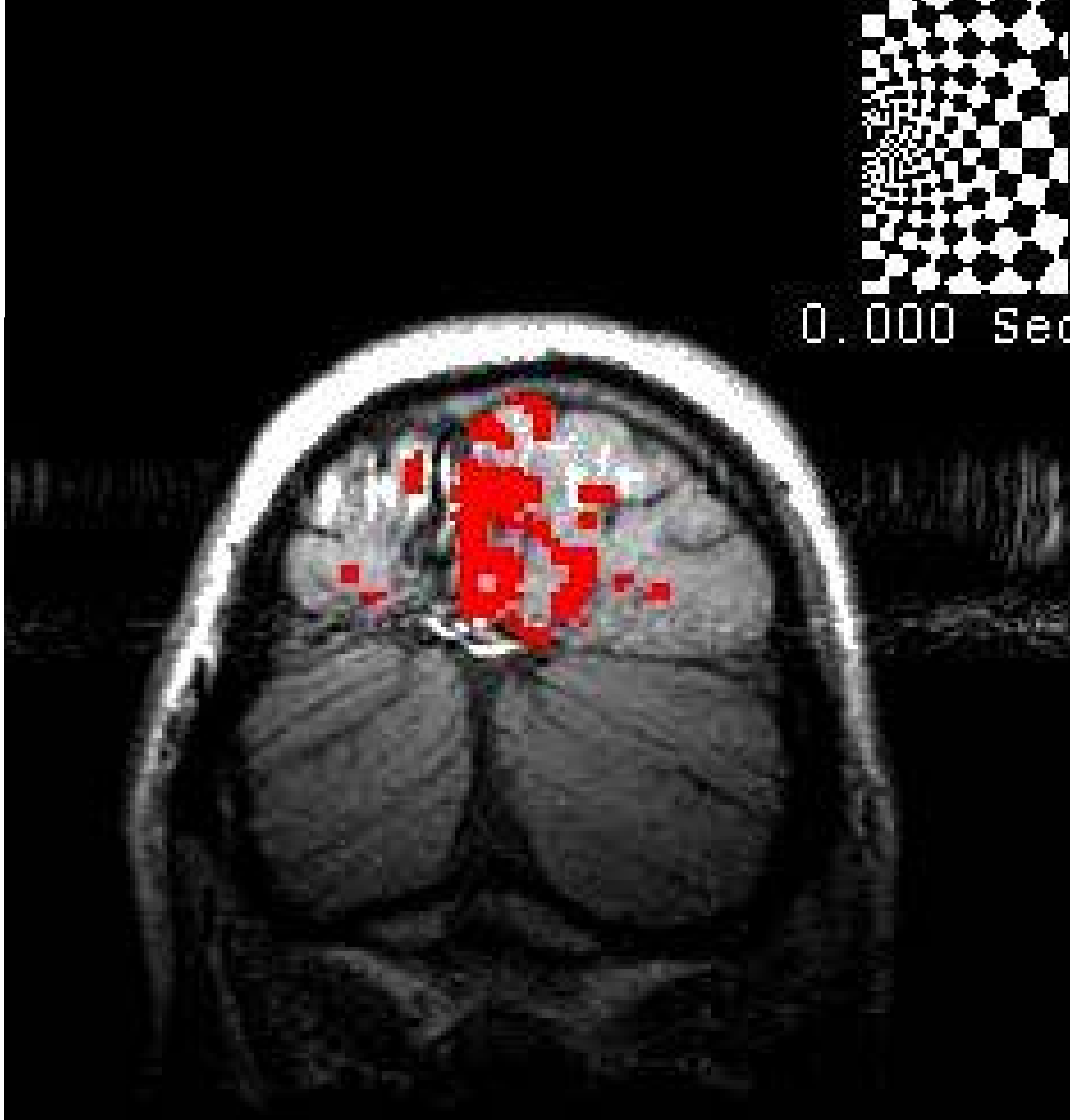
non-invasive, safe

measures Blood Oxygen Level Dependent (BOLD) response



[Mitchell et al.]





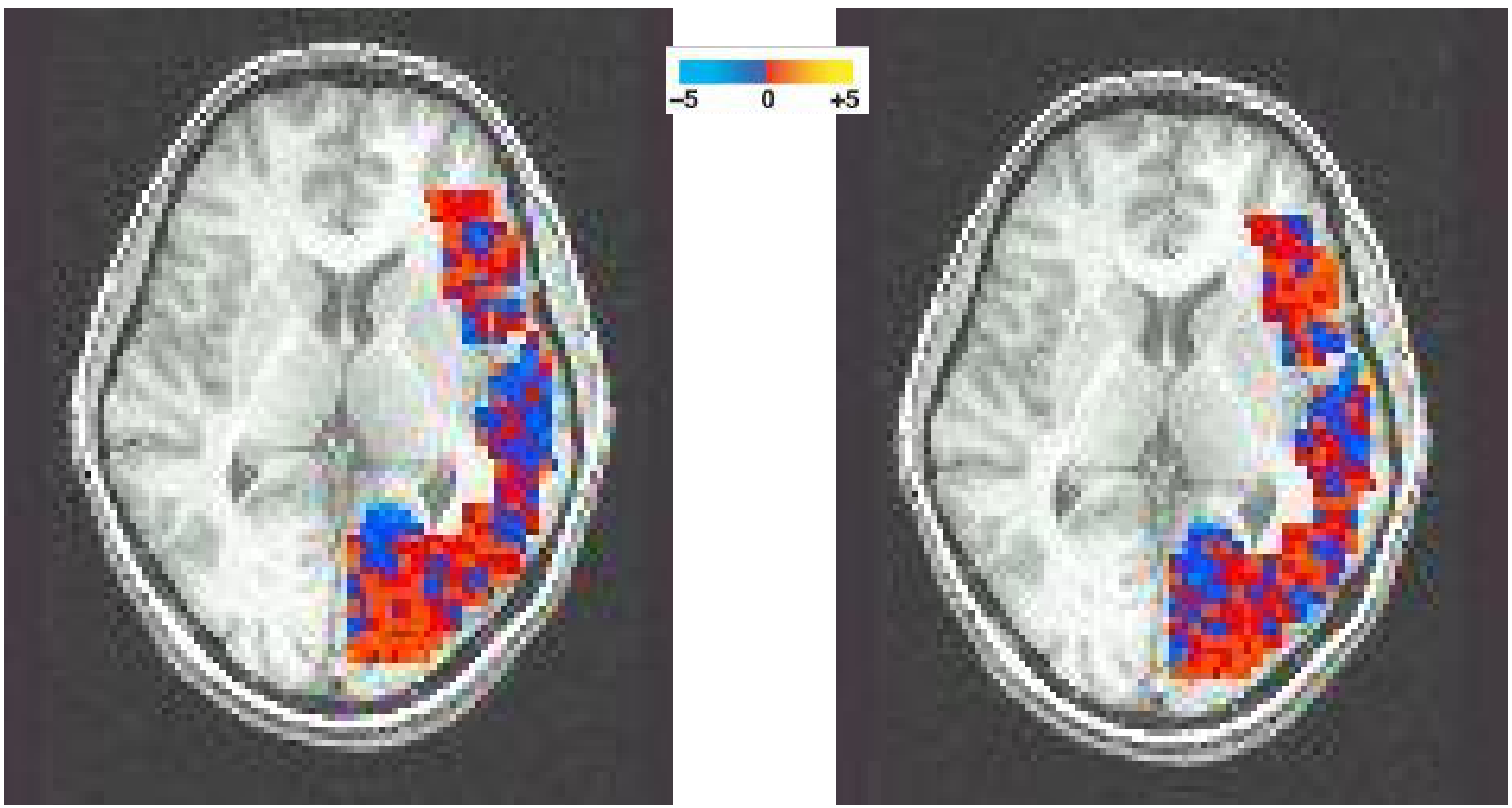
Brain scans can track activation with precision and sensitivity

slide by Barnabás Póczos & Aarti Singh

Learned Naïve Bayes Models – Means for P(BrainActivity | WordCategory)

Pairwise classification accuracy: 78-99%, 12 participants [Mitchell et al.]

Tool words | Building



slide by Barnabás Póczos & Aarti Singh

What you should know…

Naïve Bayes classifier

- What's the assumption
- Why we use it
- How do we learn it
- Why is Bayesian (MAP) estimation important

Text classification

- Bag of words model

Gaussian NB

- Features are still conditionally independent
- Each feature has a Gaussian distribution given class

slide by Barnabás Póczos & Aarti Singh