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**111010**11011111100010100010

10001011111110010110010

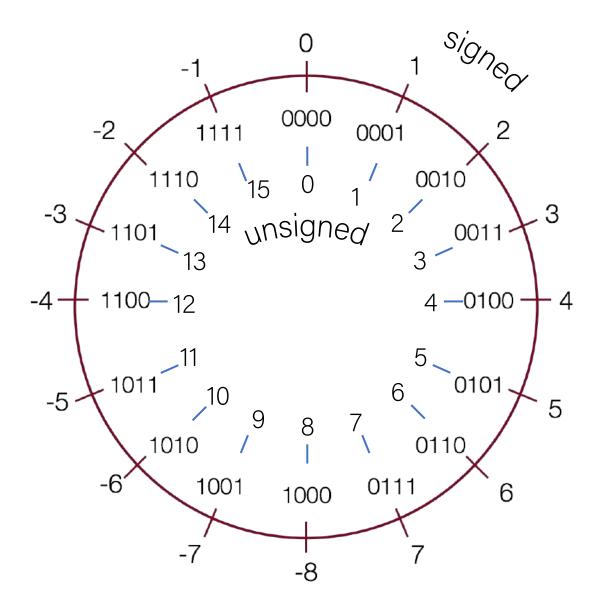
### Recap

- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Signed Integers
- Overflow
- Casting and Combining Types

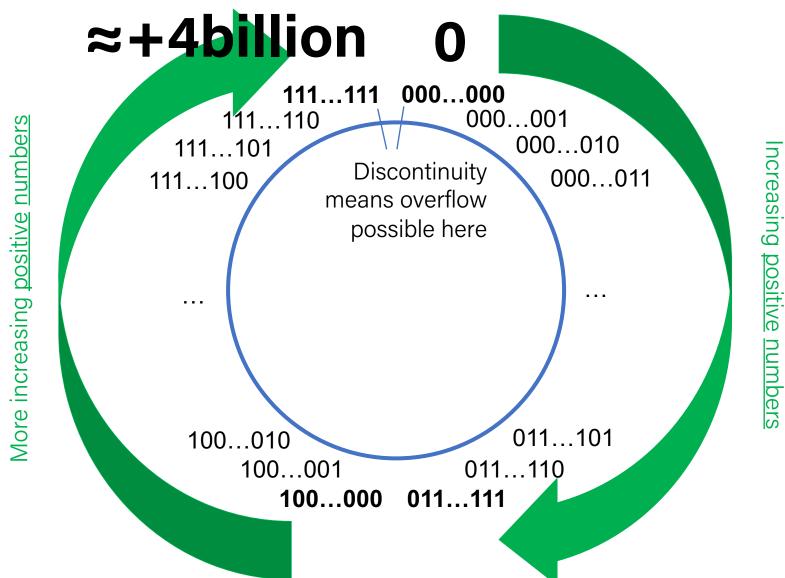
### Recap: Min and Max Integer Values

Туре	Size (Bytes)	Minimum	Maximum
char	1	-128	127
unsigned char	1	0	255
short	2	-32768	32767
unsigned short	2	0	65535
int	4	-2147483648	2147483647
unsigned int	4	0	4294967295
long	8	-9223372036854775808	9223372036854775807
unsigned long	8	0	18446744073709551615

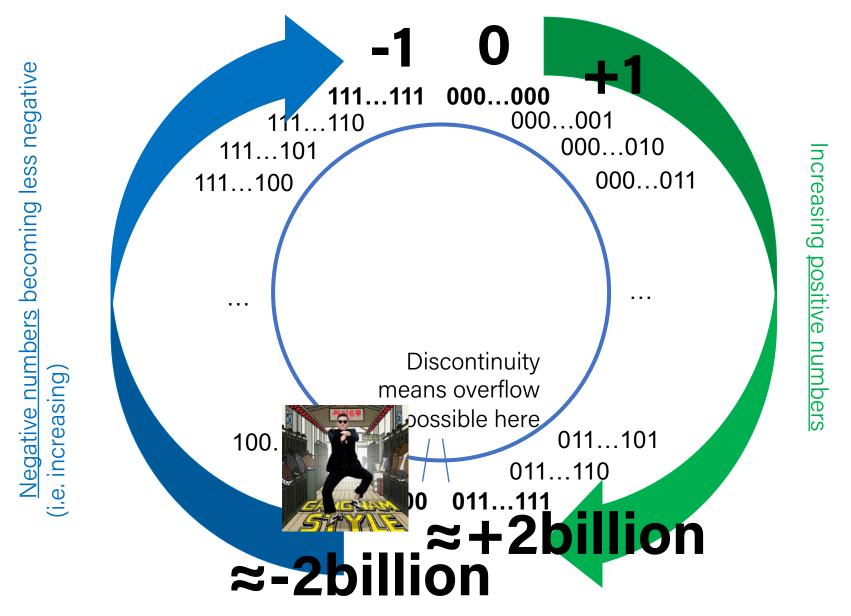
### Recap: Unsigned and Signed Integers



## Recap: Overflow in Unsigned Integers



### Recap: Overflow in Signed Numbers



### Recap: Casting

 What happens at the byte level when we cast between variable types? The bytes remain the same! This means they may be interpreted differently depending on the type.

```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);
```

This prints out: "v = -12345, uv = 4294954951".

• The bit representation for -12345 is 0b111111111111111100011111000111.

If we treat this binary representation as a positive number, it's huge!

### Bits and Bytes So Far

- All data is ultimately stored in memory in binary
- When we declare an integer variable, under the hood it is stored in binary

int x = 5; // really 0b0...0101 in memory!

- Until now, we only manipulate our integer variables in base 10 (e.g. increment, decrement, set, etc.)
- Today, we will learn about how to manipulate the underlying binary representation!
- This is useful for: more efficient arithmetic, more efficient storing of data, etc.

### Plan For Today

- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

**Disclaimer:** Slides for this lecture were borrowed from

- -Nick Troccoli's Stanford CS107 class
- -Randal E. Bryant and David R. O'Hallaron's CMU 15-213 class

### Lecture Plan

- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

### printf and Integers

- There are 3 placeholders for 32-bit integers that we can use:
  - %d: signed 32-bit int
  - %u: unsigned 32-bit int
  - %x: hex 32-bit int
- The placeholder—not the expression filling in the placeholder dictates what gets printed!



 What happens at the byte level when we cast between variable types? The bytes remain the same! This means they may be interpreted differently depending on the type.

```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);
```

This prints out: "v = -12345, uv = 4294954951". Why?



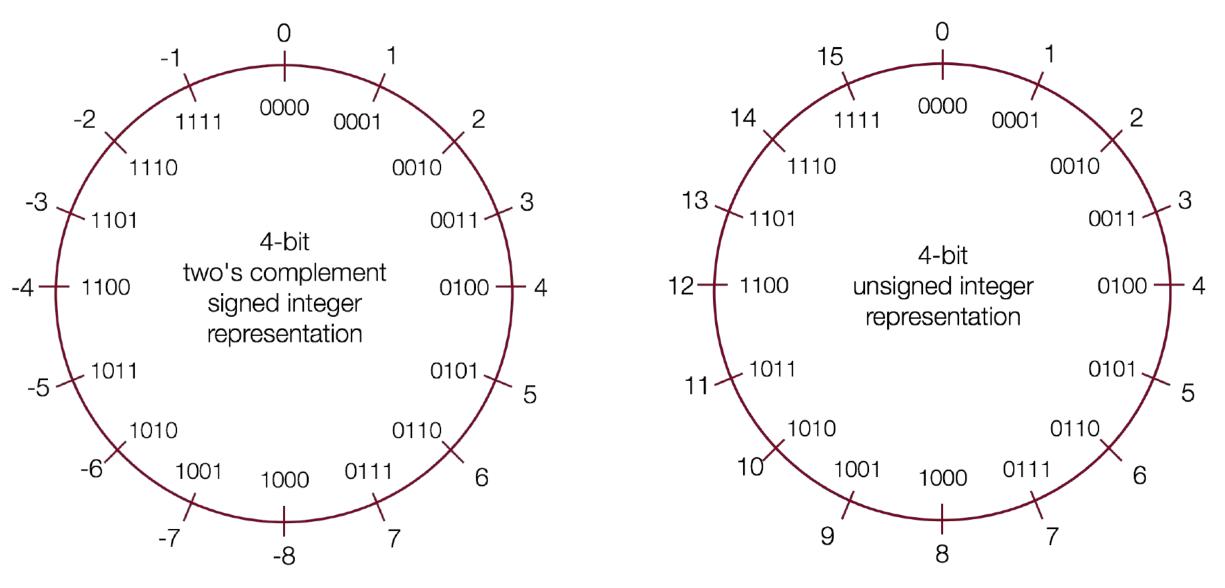
 What happens at the byte level when we cast between variable types? The bytes remain the same! This means they may be interpreted differently depending on the type.

```
int v = -12345;
unsigned int uv = v;
printf("v = %d, uv = %u\n", v, uv);
```

The bit representation for -12345 is **0b111111111111111100011111000111**.

If we treat this binary representation as a positive number, it's huge!

Casting



### Comparisons Between Different Types

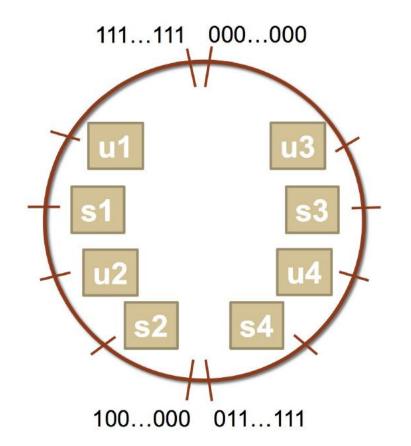
 Be careful when comparing signed and unsigned integers. C will implicitly cast the signed argument to unsigned, and then performs the operation assuming both numbers are non-negative.

Expression	Туре	Evaluation	Correct?	
0 == 0U	Unsigned	1	yes	-2 1111 0000 0001 2 1110 0010 0010
-1 < 0	Signed	1	yes	
-1 < OU	Unsigned	0	No!	-4 + 1100 0100 + 4
2147483647 > -2147483647 - 1	Signed	1	yes	-5 1011 0101 5
2147483647U > -2147483647 - 1	Unsigned	0	No!	-6 1001 1000 0111 6 -7 $-8$ 7
2147483647 > (int)2147483648U	Signed	1	No!	Type Size Minimum Maximum (Bytes)
-1 > -2	Signed	1	yes	int 4 -2147483648 2147483647
(unsigned) - 1 > -2	Unsigned	1	yes	unsigned 4 0 4294967295 int

### Comparisons Between Different Types

Which many of the following statements are true? (assume that variables are set to values that place them in the spots shown)

- s3 > u3 true
- u2 > u4 true
- s2 > s4 false
- s1 > s2 true
- u1 > u2 true
- **s1 > u3 true**



### Expanding Bit Representations

- Sometimes, we want to convert between two integers of different sizes (e.g. **short** to **int**, or **int** to **long**).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a **smaller** data type to a **bigger** data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For **signed** values, we can *repeat the sign of the value* for new digits ("sign extension")
- Note: when doing <, >, <=, >= comparison between different size types, it will promote to the larger type.

### Expanding Bit Representation

unsigned short s = 4;

```
// short is a 16-bit format, so
```

s = 0000 0000 0000 0100b

unsigned int i = s;

### Expanding Bit Representation

short s = 4;

— or —

### Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and <u>discards the *more significant bits*</u>.

int x = 53191; short sx = x; int y = sx;

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

#### 0000 0000 0000 0000 1100 1111 1100 0111

When we cast **x** to a **short**, it only has 16-bits, and C *truncates* the number:

### 1100 1111 1100 0111

This is -12345! And when we cast **sx** back an **int**, we sign-extend the number. **1111 1111 1111 1110 1100 1111 1100 0111** // still -12345

### Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and <u>discards the *more significant bits*</u>.

int x = -3; short sx = x; int y = sx;

What happens here? Let's look at the bits in x (a 32-bit int), -3:

### 1111 1111 1111 1111 1111 1111 1111 1101

When we cast **x** to a **short**, it only has 16-bits, and C *truncates* the number:

### 1111 1111 1111 1101

### Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and <u>discards the *more significant bits*</u>.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

When we cast **x** to a **short**, it only has 16-bits, and C *truncates* the number:

#### 1111 0100 0000 0000

### The sizeof Operator

```
long sizeof(type);
```

### // Example

long int\_size\_bytes = sizeof(int); // 4
long short\_size\_bytes = sizeof(short); // 2
long char\_size\_bytes = sizeof(char); // 1

**sizeof** takes a variable type as a parameter and returns the size of that type, in bytes.

What are the values of cx for the passages of code below?

short x = 130; // 0b1000 0010
char cx = x;

short x = -132 // 0b1111 1111 0111 1100
char cx = x;

short x = 25; // 0b1 1001
char cx = x;



What are the values of cx for the passages of code below?

short x = 130; // 0b0000 0000 1000 0010
char cx = x;

short x = -132 // 0b1111 1111 0111 1100
char cx = x;

short x = 25; // 0b0000 0000 0001 1001
char cx = x;



What are the values of cx for the passages of code below?

```
short x = 130; // 0b0000 0000 1000 0010
char cx = x; // -126
```

```
short x = -132 // 0b1111 1111 0111 1100
char cx = x; // 124
```

short x = 25; // 0b0000 0000 0001 1001
char cx = x; // 25



What are the values of cx for the passages of code below?

short x = 390; // 0b1 1000 0110
char cx = x;

short x = -15; // 0b1111 1111 1111 0001
char cx = x;



What are the values of cx for the passages of code below?

short x = 390; // 0b0000 0001 1000 0110
char cx = x;

short x = -15; // 0b1111 1111 1111 0001
char cx = x;



What are the values of cx for the passages of code below?

short x = 390; // 0b0000 0001 1000 0110
char cx = x; // -122

short x = -15; // 0b1111 1111 1111 0001
char cx = x; // -15



### In Sum: Basic Rules of Expanding, Truncating

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

### Lecture Plan

- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

### Number Representations

C Declaration	Intel IA32	X86-64
int	4	4
double	8	8
float	4	4
char	1	1
char *	4	8
short	2	2
long	4	8

### Byte Ordering

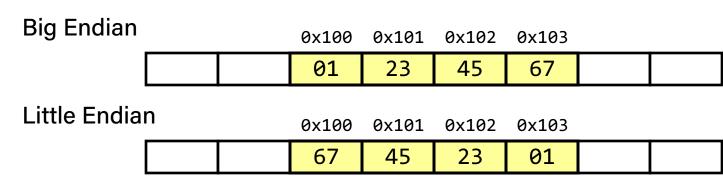
• So, how are the bytes within a multi-byte word ordered in memory?

### Byte Ordering

- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
  - -Big Endian: Sun (Oracle SPARC), PPC Mac, Internet
    - Least significant byte has highest address
  - -Little Endian: x86, ARM processors running Android, iOS, and Linux
    - Least significant byte has lowest address

## Byte Ordering Example

- Big Endian: Sun (Oracle SPARC), PPC Mac, Internet
  - Least significant byte has highest address
- Little Endian: x86, ARM processors running Android, iOS, and Linux
  - Least significant byte has lowest address
- Example:
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



# On Holy Wars and

### a Plea for Peace

Danny Cohen Information Sciences Institute

This article was written in an attempt to stop a war. I hope it is not too late for peace to prevail again. Many believe that the central question of this war is, What is the proper byte order in messages? More specifically, the question is, Which bit should travel first—the bit from the little end of the word or the bit from the big end of the word?

Followers of the former approach are called Little Endians, or Lilliputians; followers of the latter are called Big Endians, or Blefuscuians. I employ these Swiftian terms because this modern conflict is so reminiscent of the holy war described in *Gulliver's Travels*.<sup>1</sup>

#### Approaches to serialization

The above question arises as a result of the serialization process performed on messages to allow them to be sent through communication media. If the unit of communication is a message, this question has no meaning. If the units are computer words, one must determine their size and the order in which they are sent.

Since they are sent virtually at once, there is no need to determine the order of the elements of these words.

If the unit of transmission is an eight-bit byte, questions about bytes are meaningful but questions about the order of the elementary particles that constitute these bytes are not.

If the units of communication are bits, the atoms (quarks?) of computation, the only meaningful question concerns the order in which the bits are sent. Most modern communication is based on a single stream of information, the bit-stream. Hence, bits, rather than bytes or words, are the units of information that are actually transmitted over channels such as wires and satellites.

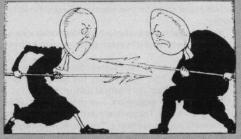
Facing page: "Audience with the Emperor of Lilliput," Gavarni, 1850's. Courtesy Library of Congress.

#### Notes on Swift's Gulliver's Travels

Swift's hero, Gulliver, is shipwrecked and washed ashore on Lilliput, whose six-inch inhabitants are required by law to break their eggs only at the little ends. Of course, all those citizens who habitually break their eggs at the big ends are angered by the proclamation. Civil war breaks out between the Little Endians and the Big Endians, resulting in the Big Endians taking refuge on a nearby island, the kingdom of Blefuscu. The controversy is ethically and politically important for the Lilliputians. In fact, Swift has 11,000 Lilliputian rebels die over the egg question. The issue might seem silly, but Swift is satirizing the actual causes of religious or holy wars.

Swift's point is that the difference between breaking an egg at the little end and breaking it at the big end is trivial. He suggests that everyone do it in his preferred way.

Of course, we are making the opposite point. We agree that the difference between sending information with the little or the big end first is trivial, but insist that everyone must do it in the same way to avoid anarchy.



Reproduced from *The Annotated Gulliver's Travels* by Isaac Asimov, Published by Crown Publishers, Inc.

0018-9162/81/1000-0048\$00.75 © 1981 IEEE

### The origin of "endian"

"Gulliver finds out that there is a law, proclaimed by the grandfather of the present ruler, requiring all citizens of Lilliput to break their eggs only at the little ends. Of course, all those citizens who broke their eggs at the big ends were angered by the proclamation. Civil war broke out between the Little-Endians and the Big-Endians, resulting in the Big-Endians taking refuge on a nearby island, the kingdom of Blefuscu."

– Danny Cohen, On Holy Wars and A Plea For Peace

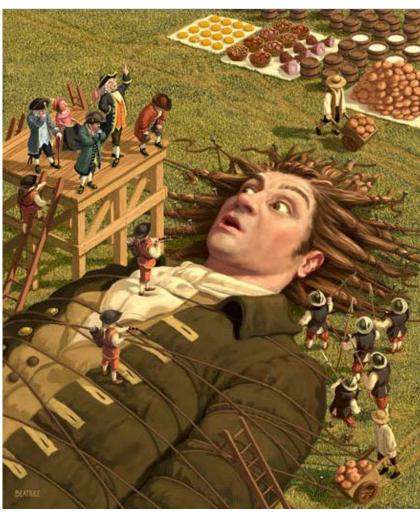
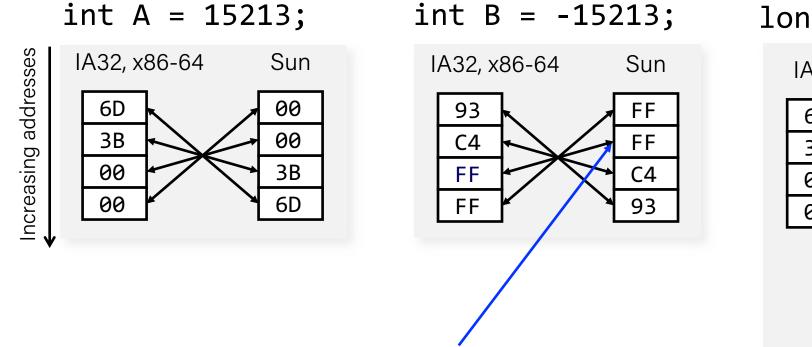


Illustration by Chris Beatrice

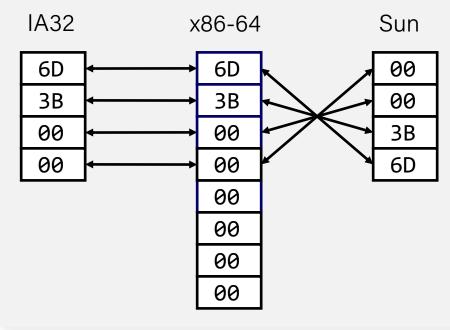
(1980)

### **Representing Integers**

Decimal:	15213	3		
Binary:	0011	1011	0110	1101
Hex:	3	В	6	D



long int C = 15213;



Two's complement representation

### Aside: ASCII

- ASCII is an encoding from common characters (letters, symbols, etc.) to bit representations (chars).
  - E.g. 'A' is 0x41
- Neat property: all uppercase letters, and all lowercase letters, are sequentially represented!
  - E.g. 'B' is 0x42

### More on this next week!

### Lecture Plan

- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

# Now that we understand binary representations, how can we manipulate them at the bit level?

### Bitwise Operators

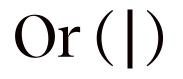
- You're already familiar with many operators in C:
  - Arithmetic operators: +, -, \*, /, %
  - Comparison operators: ==, !=, <, >, <=, >=
  - Logical Operators: &&, ||, !
- Today, we're introducing a new category of operators: bitwise operators:
   &, |, ~, ^, <<, >>



AND is a binary operator. The AND of 2 bits is 1 if both bits are 1, and 0 otherwise.

output	= a	& b;
а	b	output
0	0	0
0	1	0
1	0	0
1	1	1

& with 1 to let a bit through, & with 0 to zero out a bit



OR is a binary operator. The OR of 2 bits is 1 if either (or both) bits is 1.

out	put = a	b;
а	b	output
0	0	0
0	1	1
1	0	1
1	1	1
		1

with 1 to turn on a bit, with 0 to let a bit go through



NOT is a unary operator. The NOT of a bit is 1 if the bit is 0, or 1 otherwise.

output	= ~a;
а	output
0	1
1	0

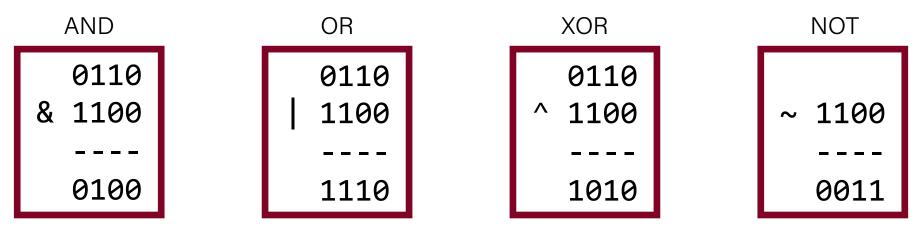
### Exclusive Or (^)

Exclusive Or (XOR) is a binary operator. The XOR of 2 bits is 1 if *exactly* one of the bits is 1, or 0 otherwise.

output	: = ;	a ^ b;
a	b	output
0	0	0
0	1	1
1	0	1
1	1	0

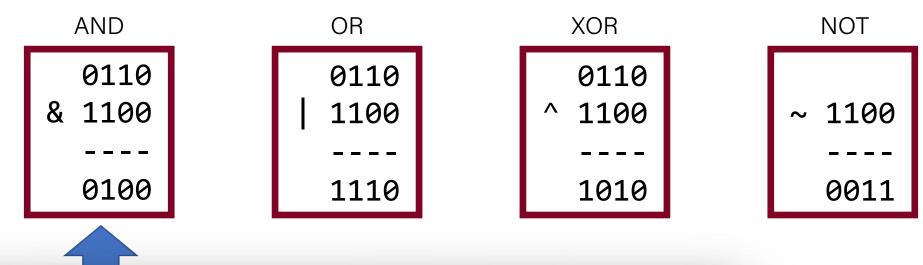
^ with 1 to flip a bit, ^ with 0 to let a bit go through

• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:



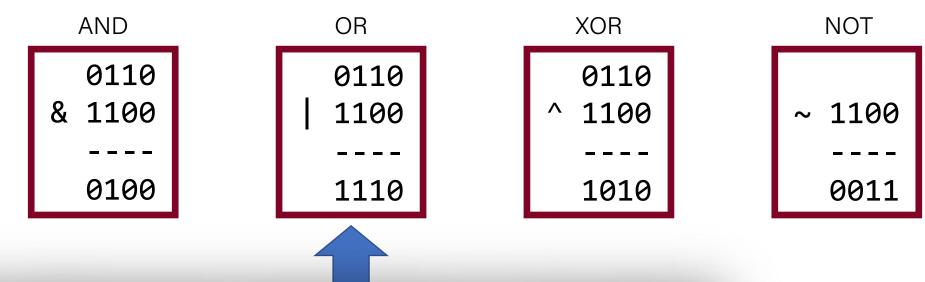
**Note:** these are different from the logical operators AND (&&), OR (||) and NOT (!).

• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:



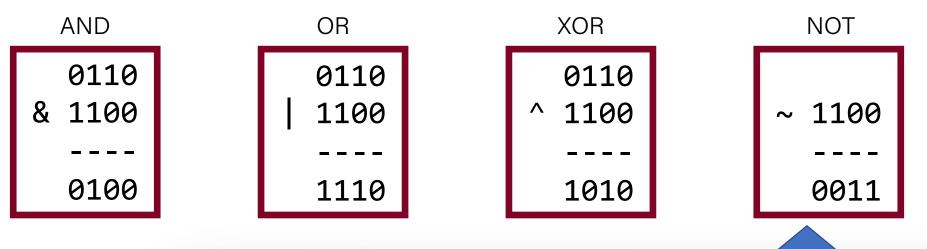
This is different from logical AND (&&). The logical AND returns true if both are nonzero, or false otherwise. With &&, this would be 6 && 12, which would evaluate to **true** (1).

• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:



This is different from logical OR (||). The logical OR returns true if either are nonzero, or false otherwise. With ||, this would be 6 || 12, which would evaluate to **true** (1).

• When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:



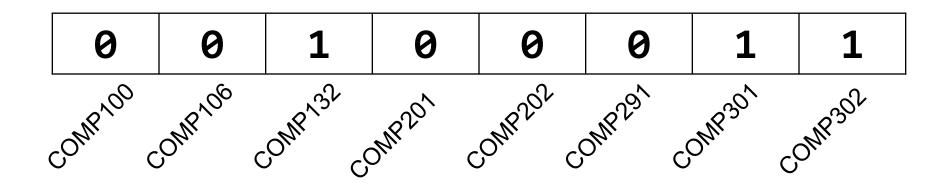
This is different from logical NOT (!). The logical NOT returns true if this is zero, and false otherwise. With !, this would be !12, which would evaluate to **false** (0).

### Lecture Plan

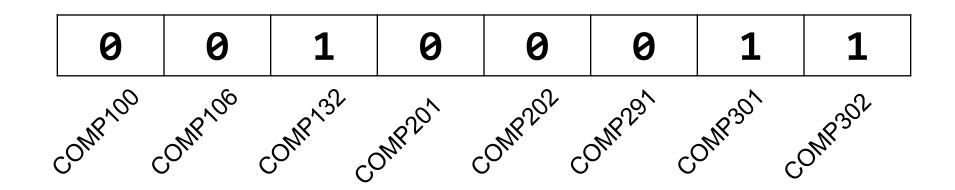
- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

### Bit Vectors and Sets

- We can use bit vectors (ordered collections of bits) to represent finite sets, and perform functions such as union, intersection, and complement.
- Example: we can represent current courses taken using a char.



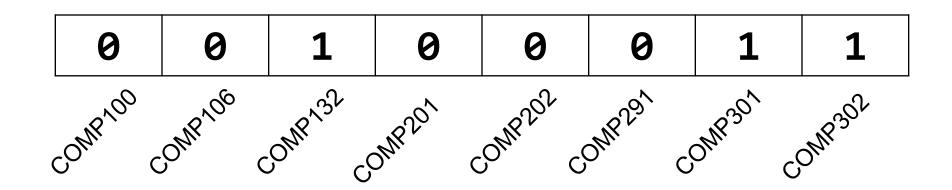
### Bit Vectors and Sets



• How do we find the union of two sets of courses taken? Use OR:

00100011 01100001 -----01100011

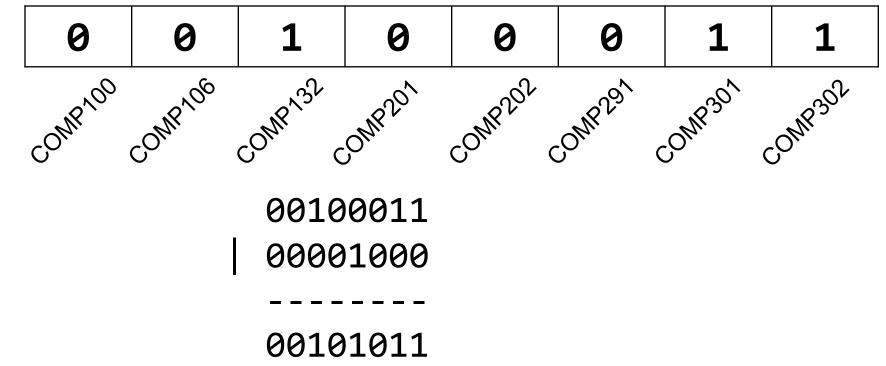
### Bit Vectors and Sets



- How do we find the intersection of two sets of courses taken? Use AND:
  - 00100011 & 01100001 -----00100001

COMP202?

- We will frequently want to manipulate or isolate out specific bits in a larger collection of bits. A **bitmask** is a constructed bit pattern that we can use, along with bit operators, to do this.
- Example: how do we update our bit vector to indicate we've taken



#define COMP100 0x1
#define COMP106 0x2
#define COMP132 0x4
#define COMP201 0x8
#define COMP202 0x10
#define COMP291 0x20
#define COMP301 0x40
#define COMP302 0x80

/*	0000	0001	*/
/*	0000	0010	*/
/*	0000	0100	*/
/*	0000	1000	*/
/*	0001	0000	*/
/*	0010	0000	*/
/*	0100	0000	*/
/*	1000	0000	*/

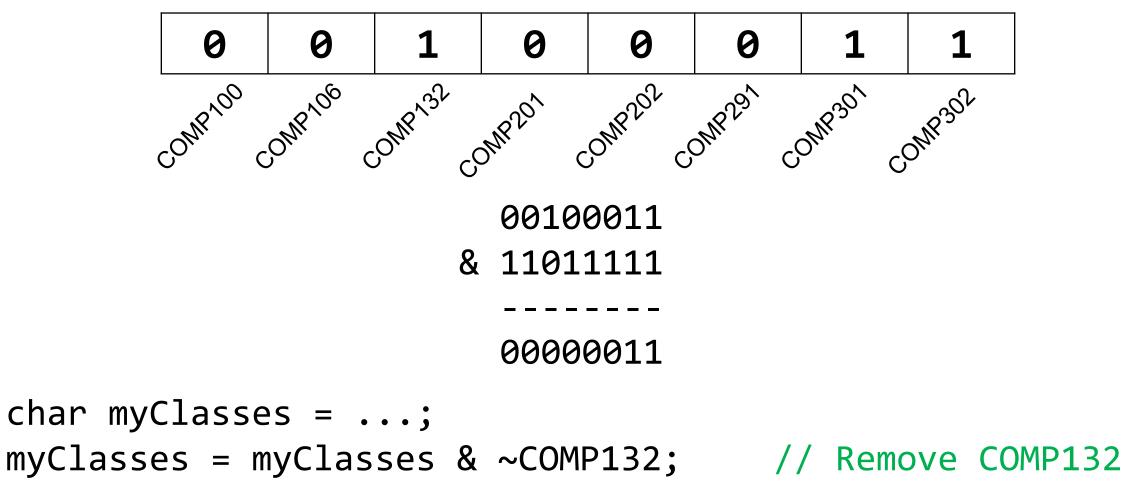
```
char myClasses = ...;
myClasses = myClasses | COMP201; // Add COMP201
```

#define COMP100 0x1
#define COMP106 0x2
#define COMP132 0x4
#define COMP201 0x8
#define COMP202 0x10
#define COMP291 0x20
#define COMP301 0x40
#define COMP302 0x80

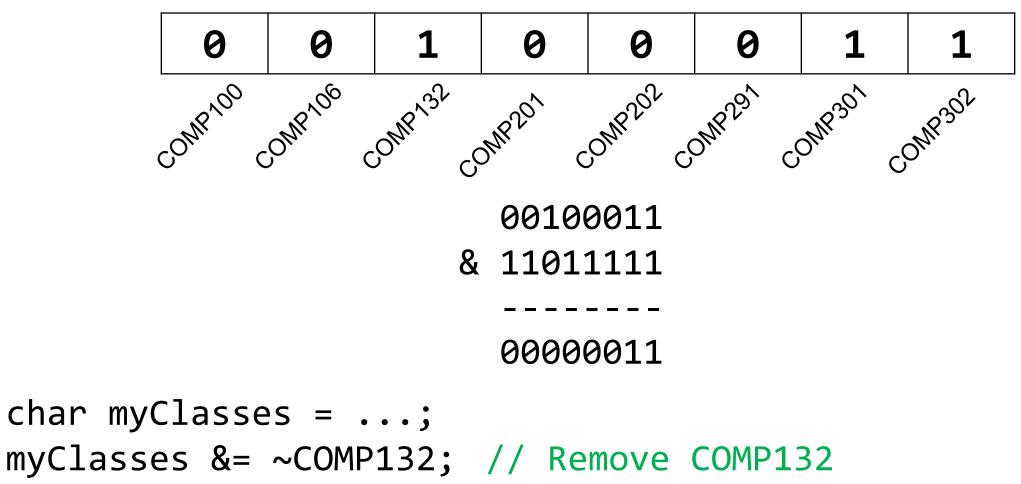
/*	0000	0001	*/
/*	0000	0010	*/
/*	0000	0100	*/
/*	0000	1000	*/
/*	0001	0000	*/
/*	0010	0000	*/
/*	0100	0000	*/
/*	1000	0000	*/

```
char myClasses = ...;
myClasses |= COMP201; // Add COMP201
```

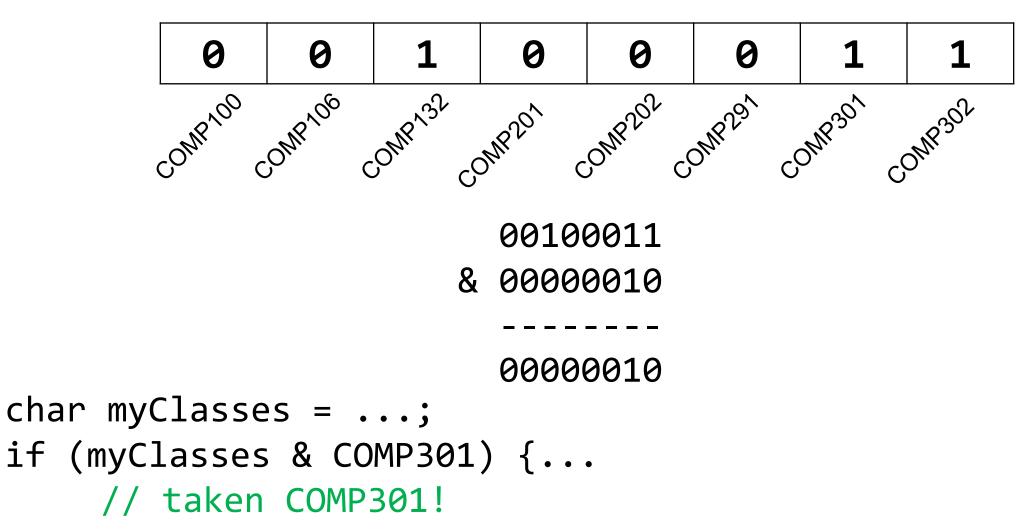
• Example: how do we update our bit vector to indicate we've *not* taken COMP132?



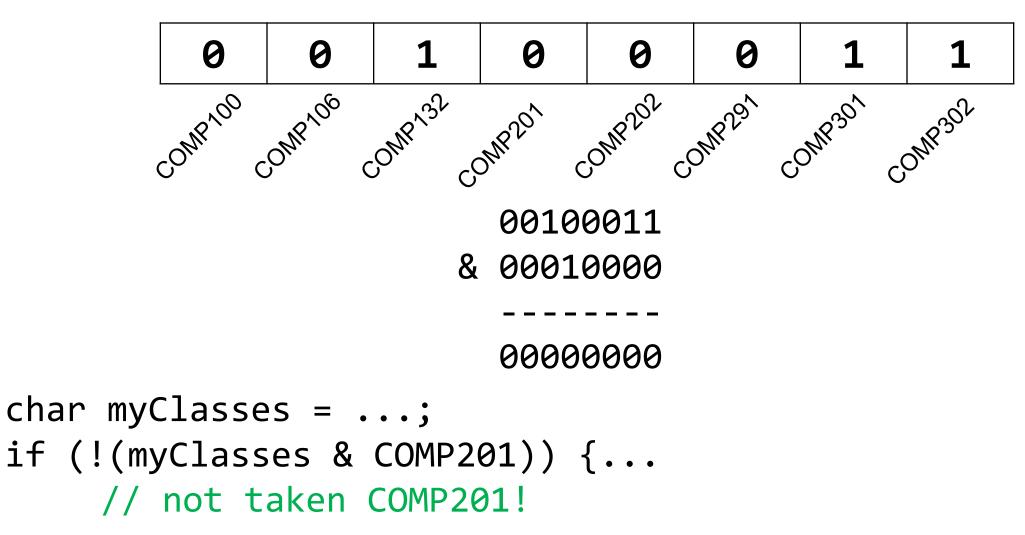
• Example: how do we update our bit vector to indicate we've *not* taken COMP132?



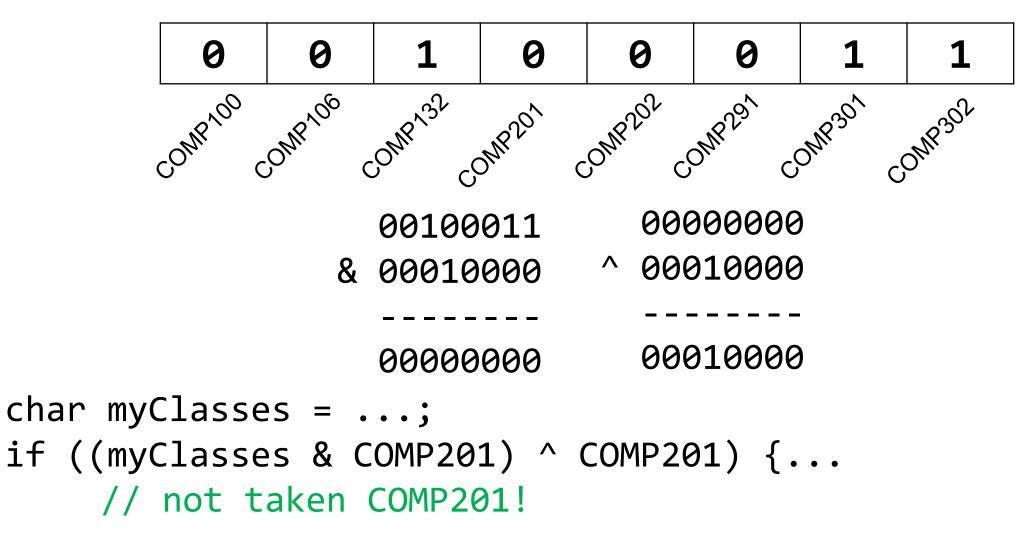
• Example: how do we check if we've taken COMP301?



• Example: how do we check if we've *not* taken COMP201?



• Example: how do we check if we've *not* taken COMP201?



### Practice: Bitwise Operations

How can we use bitmasks + bitwise operators to...

### 0b00001101

- ...turn on a particular set of bits? OR
- 2. ...turn off a particular 3. ...flip a particular set of bits? AND set of bits? XOR

0b00001101 0b00000010

0b000011<u>1</u>1

0b00001101 0b1111011

0b00001<u>0</u>01

0b00001101 0b00000110

0b00001<u>01</u>1

### Bitwise Operator Tricks

- | with 1 is useful for turning select bits on
- & with Ø is useful for turning select bits off
- | is useful for taking the union of bits
- & is useful for taking the intersection of bits
- ^ is useful for flipping select bits
- ~ is useful for flipping all bits

• Bit masking is also useful for integer representations as well. For instance, we might want to check the value of the most-significant bit, or just one of the middle bytes.

• Example: If I have a 32-bit integer j, what operation should I perform if I want to get *just the lowest byte* in j?

int j = ...; int k = j & 0xff; // mask to get just lowest byte

### Practice: Bit Masking



• **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

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Write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

(i) Start presenting to display the poll results on this slide.

### Practice: Bit Masking



- **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.
  - j | 0xff
- **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

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Write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

(i) Start presenting to display the poll results on this slide.

### Practice: Bit Masking



- **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.
  - j | 0xff
- **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.
  - j ^ ~0xff

### Powers of 2

Without using loops, how can we detect if a binary number is a power of 2? What is special about its binary representation and how can we leverage that?

# Demo: Powers of 2



Any power of 2 minus 1 is all ones:  $(2^{N} - 1 = 111...b)$ 

 $2 = 2^{1} 2 - 1 = 1$  (1b)  $4 = 2^{2} 4 - 1 = 3$  (11b)  $8 = 2^{3} 8 - 1 = 7$  (111b) Example: Take 8 1000 & 0111 = 0000

### Lecture Plan

- Casting and Combining Types (cont'd.)
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### Left Shift (<<)

The LEFT SHIFT operator shifts a bit pattern a certain number of positions to the left. New lower order bits are filled in with 0s, and bits shifted off the end are lost.

x << k; // evaluates to x shifted to the left by k bits
x <<= k; // shifts x to the left by k bits</pre>

8-bit examples:

00110111 << 2 results in 11011100 01100011 << 4 results in 00110000 10010101 << 4 results in 01010000

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

x >> k; // evaluates to x shifted to the right by k bit x >>= k; // shifts x to the right by k bits

**Question:** how should we fill in new higher-order bits? **Idea:** let's follow left-shift and fill with 0s.

short x = 2; // 0000 0000 0000 0010
x >>= 1; // 0000 0000 0000 0001
printf("%d\n", x); // 1

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

x >> k; // evaluates to x shifted to the right by k bit x >>= k; // shifts x to the right by k bits

**Question:** how should we fill in new higher-order bits? **Idea:** let's follow left-shift and fill with 0s.

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

x >> k; // evaluates to x shifted to the right by k bit x >>= k; // shifts x to the right by k bits

**Question**: how should we fill in new higher-order bits? **Problem**: always filling with zeros means we may change the sign bit. **Solution**: let's fill with the sign bit!

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

x >> k; // evaluates to x shifted to the right by k bit x >>= k; // shifts x to the right by k bits

**Question:** how should we fill in new higher-order bits? **Solution:** let's fill with the sign bit!

short x = 2; // 0000 0000 0000 0010
x >>= 1; // 0000 0000 0000 0001
printf("%d\n", x); // 1

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

x >> k; // evaluates to x shifted to the right by k bit x >>= k; // shifts x to the right by k bits

**Question:** how should we fill in new higher-order bits? **Solution:** let's fill with the sign bit!

short x = -2; // 1111 1111 1111 1110
x >>= 1; // 1111 1111 1111 1111
printf("%d\n", x); // -1!

There are *two kinds* of right shifts, depending on the value and type you are shifting:

- Logical Right Shift: fill new high-order bits with 0s.
- Arithmetic Right Shift: fill new high-order bits with the most-significant bit.

Unsigned numbers are right-shifted using Logical Right Shift. Signed numbers are right-shifted using Arithmetic Right Shift.

This way, the sign of the number (if applicable) is preserved!

### Shift Operation Pitfalls

- Technically, the C standard does not precisely define whether a right shift for signed integers is logical or arithmetic. However, almost all compilers/machines use arithmetic, and you can most likely assume this.
- 2. Operator precedence can be tricky! For example:

**1<<2 + 3<<4** means **1 << (2+3) << 4** because addition and subtraction have higher precedence than shifts! Always use parentheses to be sure:

(1<<2) + (3<<4)

### Bit Operator Pitfalls

- The default type of a number literal in your code is an **int**.
- Let's say you want a long with the index-32 bit as 1:

### long num = 1 << 32;

This doesn't work! 1 is by default an **int**, and you can't shift an **int** by 32 because it only has 32 bits. You must specify that you want 1 to be a **long**.

long num = 1L << 32;

### Recap

- Casting and Combining Types (cont'd.)
- Byte Ordering
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

**Next time:** *More on how can a computer represent floating point numbers?*