photo by unsplash user @swimstaralex

COMP201

Computer Systems & Programming

Lecture #04 – Bits and Bitwise Operators

KOÇ UNIVERSITY

Aykut Erdem // Koç University // Fall 2020

Last Time

- Signed Integers
- Overflow
- Casting and Combining Types

Plan For Today

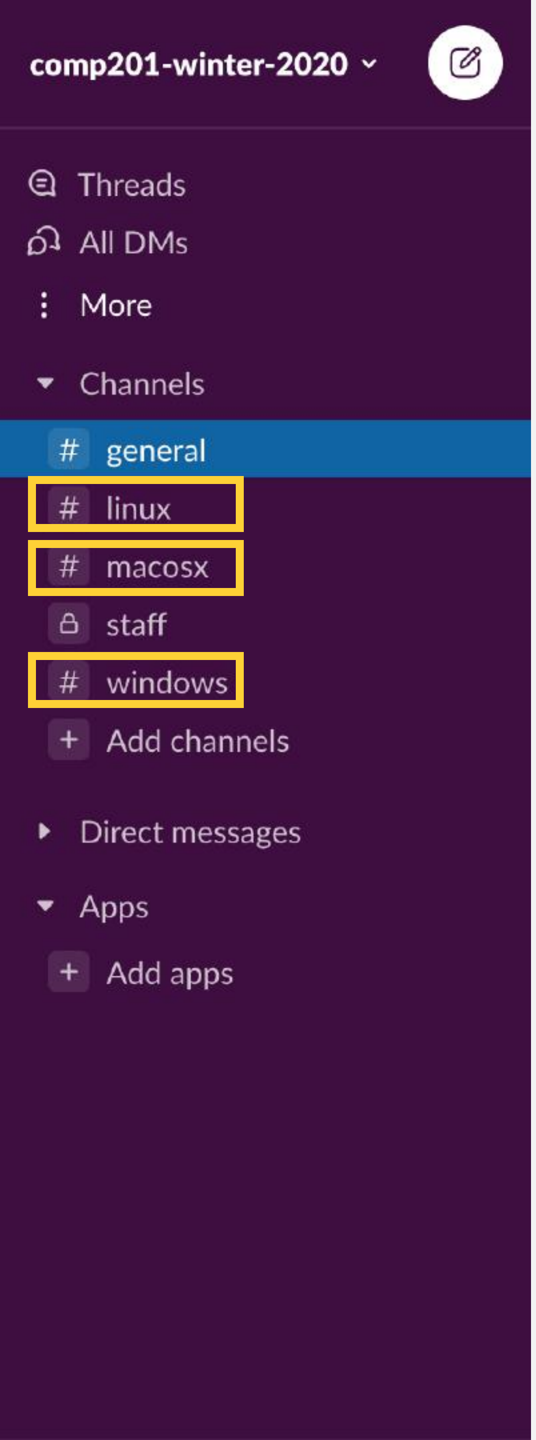
- Casting and Combining Types (cont'd.)
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Disclaimer: Slides for this lecture were borrowed from
—Nick Troccoli's Stanford CS107 class

Image: Professor Farnsworth (Futurama)

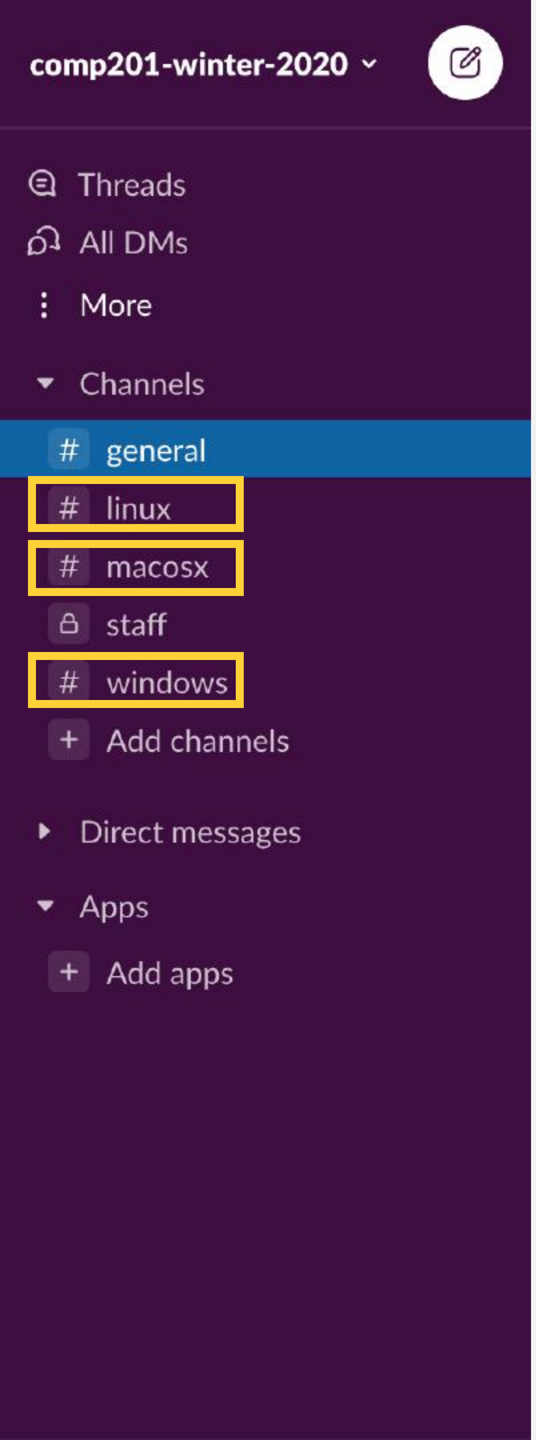
Good news, everyone!

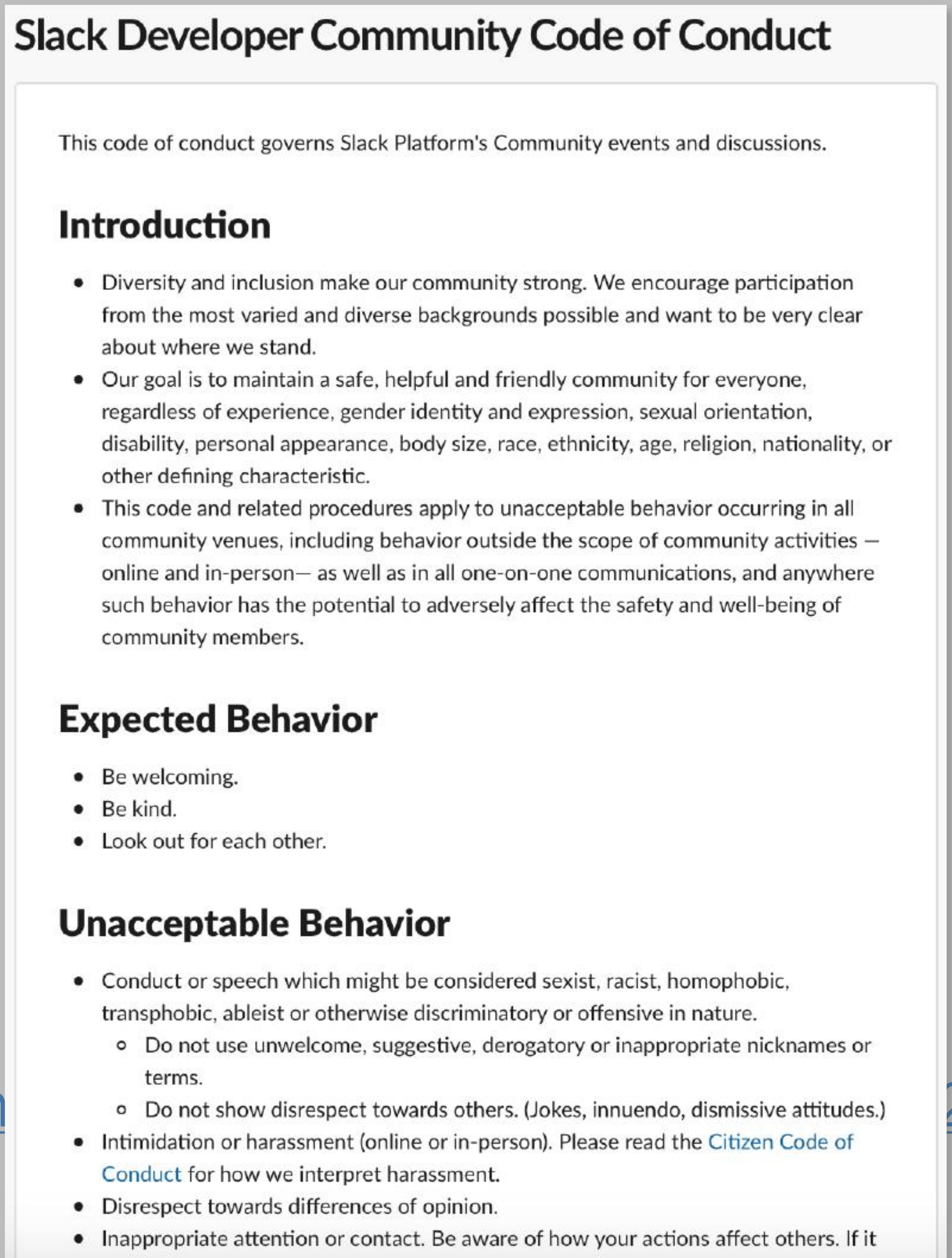
- Labs start this week. Check your section assignment.
- Assg1 will be out on Oct 16 (due Oct 26)
- From now on, I will be having my office hour every Thursday from 2pm to 3pm.



New: COMP201 Slack Workspace

https://join.slack.com/t/comp201-winter-2020/signup





Slack Developer Community Code of Conduct

This code of conduct governs Slack Platform's Community events and discussions.

Introduction

- Diversity and inclusion make our community strong. We encourage participation from the most varied and diverse backgrounds possible and want to be very clear about where we stand.
- Our goal is to maintain a safe, helpful and friendly community for everyone, regardless of experience, gender identity and expression, sexual orientation, disability, personal appearance, body size, race, ethnicity, age, religion, nationality, or other defining characteristic.
- This code and related procedures apply to unacceptable behavior occurring in all community venues, including behavior outside the scope of community activities — online and in-person— as well as in all one-on-one communications, and anywhere such behavior has the potential to adversely affect the safety and well-being of community members.

Expected Behavior

- Be welcoming.
- Be kind.
- Look out for each other.

Unacceptable Behavior

- Conduct or speech which might be considered sexist, racist, homophobic, transphobic, ableist or otherwise discriminatory or offensive in nature.
  - Do not use unwelcome, suggestive, derogatory or inappropriate nicknames or terms.
  - Do not show disrespect towards others. (Jokes, innuendo, dismissive attitudes.)
- Intimidation or harassment (online or in-person). Please read the Citizen Code of Conduct for how we interpret harassment.
- Disrespect towards differences of opinion.
- Inappropriate attention or contact. Be aware of how your actions affect others. If it

https://join ... 20/signup

Lecture Plan

- Casting and Combining Types (cont'd.)
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Expanding Bit Representations

- Sometimes, we want to convert between two integers of different sizes (e.g. **`short`** to **`int`**, or **`int`** to **`long`**).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a **smaller** data type to a **bigger** data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For **signed** values, we can *repeat the sign of the value* for new digits ("sign extension")
- Note: when doing <, >, <=, >= comparison between different size types, it will *promote to the larger type*.

Expanding Bit Representation

```
unsigned short s = 4;
// short is a 16-bit format, so                          s = 0000 0000 0000 0100b

unsigned int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

Expanding Bit Representation

```
short s = 4;
// short is a 16-bit format, so                     s = 0000 0000 0000 0100b

int i = s;
// conversion to 32-bit int, so i = 0000 0000 0000 0000 0000 0000 0000 0100b
```

— or —

```
short s = -4;
// short is a 16-bit format, so                     s = 1111 1111 1111 1100b

int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 1111 1100b
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

`0000 0000 0000 0000 1100 1111 1100 0111`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1100 1111 1100 0111`

This is -12345! And when we cast sx back an int, we sign-extend the number.

`1111 1111 1111 1111 1100 1111 1100 0111` // still -12345

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

`1111 1111 1111 1111 1111 1111 1111 1101`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1111 1111 1111 1101`

This is -3! **If the number does fit, it will convert fine.** y looks like this:

`1111 1111 1111 1111 1111 1111 1111 1101` // still -3

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

`0000 0000 0000 0001 1111 0100 0000 0000`

When we cast x to a short, it only has 16-bits, and C *truncates* the number:

`1111 0100 0000 0000`

This is 62464! **Unsigned numbers can lose info too.** Here is what y looks like:

`0000 0000 0000 0000 1111 0100 0000 0000` // still 62464

The `sizeof` Operator

```
long sizeof(type);
```

```
// Example
long int_size_bytes = sizeof(int);     // 4
long short_size_bytes = sizeof(short); // 2
long char_size_bytes = sizeof(char);   // 1
```

`sizeof` takes a variable type as a parameter and returns the size of that type, in bytes.

Bits and Bytes So Far

- all data is ultimately stored in memory in binary
- When we declare an integer variable, under the hood it is stored in binary

```
int x = 5;// really 0b0…0101 in memory!
```

- Until now, we only manipulate our integer variables in base 10 (e.g. increment, decrement, set, etc.)
- Today, we will learn about how to manipulate the underlying binary representation!
- This is useful for: more efficient arithmetic, more efficient storing of data, etc.

Aside: ASCII

- ASCII is an encoding from common characters (letters, symbols, etc.) to bit representations (chars).
  - E.g. 'A’ is 0x41
- Neat property: all uppercase letters, and all lowercase letters, are sequentially represented!
  - E.g. 'B’ is 0x42

Lecture Plan

- Casting and Combining Types (cont'd.)
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Now that we understand binary representations, how can we manipulate them at the bit level?

Bitwise Operators

- You're already familiar with many operators in C:
  - **Arithmetic operators:** `+`, `-`, `*`, `/`, `%`
  - **Comparison operators**: `==`, `!=`, `<`, `>`, `<=`, `>=`
  - **Logical Operators**: `&&`, `||`, `!`

- Today, we're introducing a new category of operators: **bitwise operators:**
  - `&`, `|`, `~`, `^`, `<<`, `>>`

And (&)

AND is a binary operator. The AND of 2 bits is 1 if both bits are 1, and 0 otherwise.

```
output = a & b;
```

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

`&` with `1` to let a bit through, `&` with `0` to zero out a bit

Or (|)

OR is a binary operator. The OR of 2 bits is 1 if either (or both) bits is 1.

```
output = a | b;
```

a	b	output
0	0	0
0	1	1
1	0	1
1	1	1

| with `1` to turn on a bit, | with `0` to let a bit go through

Not (~)

NOT is a unary operator. The NOT of a bit is 1 if the bit is 0, or 1 otherwise.

```
output = ~a;
```

a	output
0	1
1	0

Exclusive Or (^)

Exclusive Or (XOR) is a binary operator. The XOR of 2 bits is 1 if *exactly* one of the bits is 1, or 0 otherwise.

```
output = a ^ b;
```

a	**b**	**output**
0	0	0
0	1	1
1	0	1
1	1	0

^ with `1` to flip a bit, ^ with `0` to let a bit go through

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
  0110
& 1100
  ----
  0100
```

OR

```
  0110
| 1100
  ----
  1110
```

XOR

```
  0110
^ 1100
  ----
  1010
```

NOT

```
~ 1100
  ----
  0011
```

Note: these are different from the logical operators AND (&&), OR (||) and NOT (!).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND	OR	XOR	NOT

`  0110`
`& 1100`
`  ----`
`  0100`
	`  0110`
	`| 1100`
	`  ----`
	`  1110`
		`  0110`
		`^ 1100`
		`  ----`
		`  1010`
			`~ 1100`
			`  ----`
			`  0011`

This is different from logical AND (&&). The logical AND returns true if both are nonzero, or false otherwise. With &&, this would be `6 && 12`, which would evaluate to **true** (1).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND
```
  0110
& 1100
  ----
  0100
```

OR
```
  0110
| 1100
  ----
  1110
```

XOR
```
  0110
^ 1100
  ----
  1010
```

NOT
```
~ 1100
  ----
  0011
```

This is different from logical OR (||). The logical OR returns true if either are nonzero, or false otherwise. With ||, this would be `6 || 12`, which would evaluate to **true** (1).

Operators on Multiple Bits

- When these operators are applied to numbers (multiple bits), the operator is applied to the corresponding bits in each number. For example:

AND

```
  0110
& 1100
  ----
  0100
```

OR

```
  0110
| 1100
  ----
  1110
```

XOR

```
  0110
^ 1100
  ----
  1010
```

NOT

```

~ 1100
  ----
  0011
```

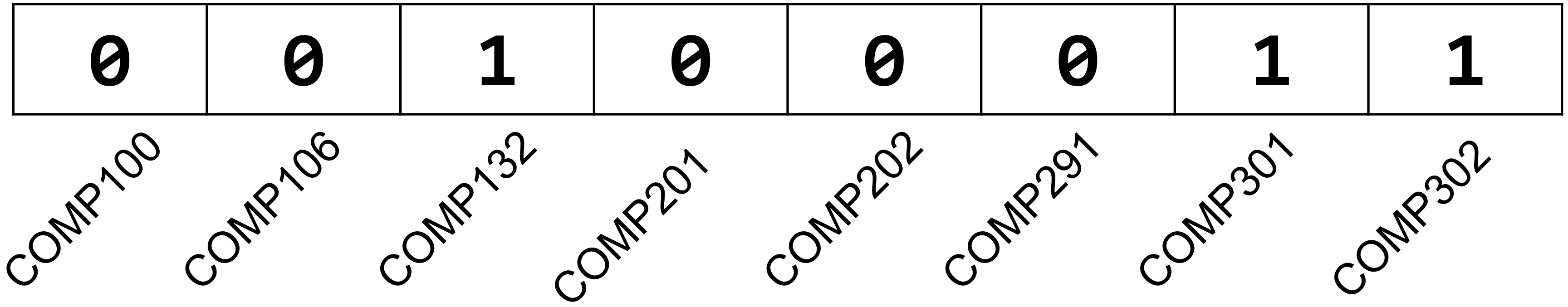
This is different from logical NOT (`!`). The logical NOT returns true if this is zero, and false otherwise. With `!`, this would be `!12`, which would evaluate to **false** (`0`).

Lecture Plan

- Casting and Combining Types (cont'd.)
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Bit Vectors and Sets

- We can use bit vectors (ordered collections of bits) to represent finite sets, and perform functions such as union, intersection, and complement.
- **Example:** we can represent current courses taken using a **char**.



Bit Vectors and Sets

0	0	1	0	0	0	1	1
COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302

- How do we find the union of two sets of courses taken? Use OR:

```
  00100011
| 01100001
  --------
  01100011
```

Bit Vectors and Sets

COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302
0	0	1	0	0	0	1	1

- How do we find the intersection of two sets of courses taken? Use AND:

```
  00100011
& 01100001
  --------
  00100001
```

Bit Masking

- We will frequently want to manipulate or isolate out specific bits in a larger collection of bits. A **bitmask** is a constructed bit pattern that we can use, along with bit operators, to do this.
- **Example:** how do we update our bit vector to indicate we've taken COMP202?

0	0	1	0	0	0	1	1
COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302

```
  00100011
| 00001000
  --------
  00101011
```

Bit Masking

```
#define COMP100 0x1      /* 0000 0001 */
#define COMP106 0x2      /* 0000 0010 */
#define COMP132 0x4      /* 0000 0100 */
#define COMP201  0x8     /* 0000 1000 */
#define COMP202  0x10    /* 0001 0000 */
#define COMP291  0x20    /* 0010 0000 */
#define COMP301  0x40    /* 0100 0000 */
#define COMP302  0x80    /* 1000 0000 */

char myClasses = ...;
myClasses = myClasses | COMP201; // Add COMP201
```

Bit Masking

```
#define COMP100 0x1       /* 0000 0001 */
#define COMP106 0x2       /* 0000 0010 */
#define COMP132 0x4       /* 0000 0100 */
#define COMP201  0x8      /* 0000 1000 */
#define COMP202  0x10     /* 0001 0000 */
#define COMP291  0x20     /* 0010 0000 */
#define COMP301  0x40     /* 0100 0000 */
#define COMP302  0x80     /* 1000 0000 */

char myClasses = ...;
myClasses |= COMP201;   // Add COMP201
```

Bit Masking

- **Example:** how do we update our bit vector to indicate we've *not* taken COMP132?

COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302
0	0	1	0	0	0	1	1

```
  00100011
& 11011111
  --------
  00000011
```

```
char myClasses = ...;
myClasses = myClasses & ~COMP132;      // Remove COMP132
```

Bit Masking

- **Example:** how do we update our bit vector to indicate we've *not* taken COMP132?

COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302
0	0	1	0	0	0	1	1

```
  00100011
& 11011111
  --------
  00000011
```

```
char myClasses = ...;
myClasses &= ~COMP132;  // Remove COMP132
```

Bit Masking

- **Example:** how do we check if we've taken COMP301?

0	0	1	0	0	0	1	1
COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302

```
  00100011
& 00000010
  --------
  00000010
```

```
char myClasses = ...;
if (myClasses & COMP301) {...
    // taken COMP301!
```

Bit Masking

- **Example:** how do we check if we've *not* taken COMP201?

COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302
0	0	1	0	0	0	1	1

```
  00100011
& 00010000
  --------
  00000000
```

```
char myClasses = ...;
if (!(myClasses & COMP201)) {...
     // not taken COMP201!
```

Bit Masking

- **Example:** how do we check if we've *not* taken COMP201?

COMP100	COMP106	COMP132	COMP201	COMP202	COMP291	COMP301	COMP302
0	0	1	0	0	0	1	1

```
  00100011      00000000
& 00010000    ^ 00010000
  ---------     ---------
  00000000      00001000
```

```
char myClasses = ...;
if ((myClasses & COMP201) ^ COMP201) {...
    // not taken COMP201!
```

Bitwise Operator Tricks

- `|` with `1` is useful for turning select bits on
- `&` with `0` is useful for turning select bits off
- `|` is useful for taking the union of bits
- `&` is useful for taking the intersection of bits
- `^` is useful for flipping select bits
- `~` is useful for flipping all bits

Bit Masking

- Bit masking is also useful for integer representations as well. For instance, we might want to check the value of the most-significant bit, or just one of the middle bytes.

- **Example**: If I have a 32-bit integer **`j`**, what operation should I perform if I want to get *just the lowest byte* in **`j`**?

```
int j = ...;
int k = j & 0xff;  // mask to get just lowest byte
```

Practice: Bit Masking

- **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

- **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

Practice: Bit Masking

- **Practice 1:** write an expression that, given a 32-bit integer j, sets its least-significant byte to all 1s, but preserves all other bytes.

```
j | 0xff
```

- **Practice 2:** write an expression that, given a 32-bit integer j, flips ("complements") all but the least-significant byte, and preserves all other bytes.

```
j ^ ~0xff
```

Powers of 2

Without using loops, how can we detect if a binary number is a power of 2? What is special about its binary representation and how can we leverage that?

Demo: Powers of 2

Lecture Plan

- Casting and Combining Types (cont'd.)
- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Left Shift (<<)

The LEFT SHIFT operator shifts a bit pattern a certain number of positions to the left. New lower order bits are filled in with 0s, and bits shifted off the end are lost.

```
x << k;     // evaluates to x shifted to the left by k bits
x <<= k;    // shifts x to the left by k bits
```

8-bit examples:

```
00110111 << 2 results in 11011100
01100011 << 4 results in 00110000
10010101 << 4 results in 01010000
```

Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

```
 x >> k;     // evaluates to x shifted to the right by k bits
x >>= k;     // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Idea: let's follow left-shift and fill with 0s.

```
short x = 2;  // 0000 0000 0000 0010
x >>= 1;      // 0000 0000 0000 0001
printf("%d\n", x); // 1
```

Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

```
 x >> k;     // evaluates to x shifted to the right by k bit
x >>= k;     // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Idea: let's follow left-shift and fill with 0s.

```
short x = -2; // 1111 1111 1111 1110
x >>= 1;      // 0111 1111 1111 1111
printf("%d\n", x); // 32767!
```

Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

```
 x >> k;     // evaluates to x shifted to the right by k bit
x >>= k;     // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Problem: always filling with zeros means we may change the sign bit.

Solution: let's fill with the sign bit!

Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

```
x >> k;     // evaluates to x shifted to the right by k bit
x >>= k;    // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Solution: let's fill with the sign bit!

```
short x = 2;   // 0000 0000 0000 0010
x >>= 1;       // 0000 0000 0000 0001
printf("%d\n", x); // 1
```

Right Shift (>>)

The RIGHT SHIFT operator shifts a bit pattern a certain number of positions to the right. Bits shifted off the end are lost.

```
x >> k;     // evaluates to x shifted to the right by k bit
x >>= k;    // shifts x to the right by k bits
```

Question: how should we fill in new higher-order bits?

Solution: let's fill with the sign bit!

```
short x = -2; // 1111 1111 1111 1110
x >>= 1;      // 1111 1111 1111 1111
printf("%d\n", x); // -1!
```

Right Shift (>>)

There are *two kinds* of right shifts, depending on the value and type you are shifting:

- **Logical Right Shift:** fill new high-order bits with 0s.
- **Arithmetic Right Shift:** fill new high-order bits with the most-significant bit.

Unsigned numbers are right-shifted using **Logical Right Shift**.

Signed numbers are right-shifted using **Arithmetic Right Shift.**

This way, the sign of the number (if applicable) is preserved!

Shift Operation Pitfalls

1. *Technically*, the C standard does not precisely define whether a right shift for signed integers is logical or arithmetic. However, **almost all compilers/machines** use arithmetic, and you can most likely assume this.
2. Operator precedence can be tricky! For example:

    `1<<2 + 3<<4` means **`1 << (2+3) << 4`** because addition and subtraction have higher precedence than shifts! Always use parentheses to be sure:

    `(1<<2) + (3<<4)`

Bit Operator Pitfalls

- The default type of a number literal in your code is an **int**.
- Let's say you want a long with the index-32 bit as 1:

```
long num = 1 << 32;
```

- This doesn't work! 1 is by default an **int**, and you can't shift an int by 32 because it only has 32 bits. You must specify that you want 1 to be a **long**.

```
long num = 1L << 32;
```

Recap

- Bitwise Operators
- Bitmasks
- Bit Shift Operators

Next time: *How can a computer represent floating point numbers?*