Video: Samples from "cooking" subset of Kinetics, Weissenborn et al.

COMP547

DEEP UNSUPERVISED LEARNING

Lecture #5 – Autoregressive Models

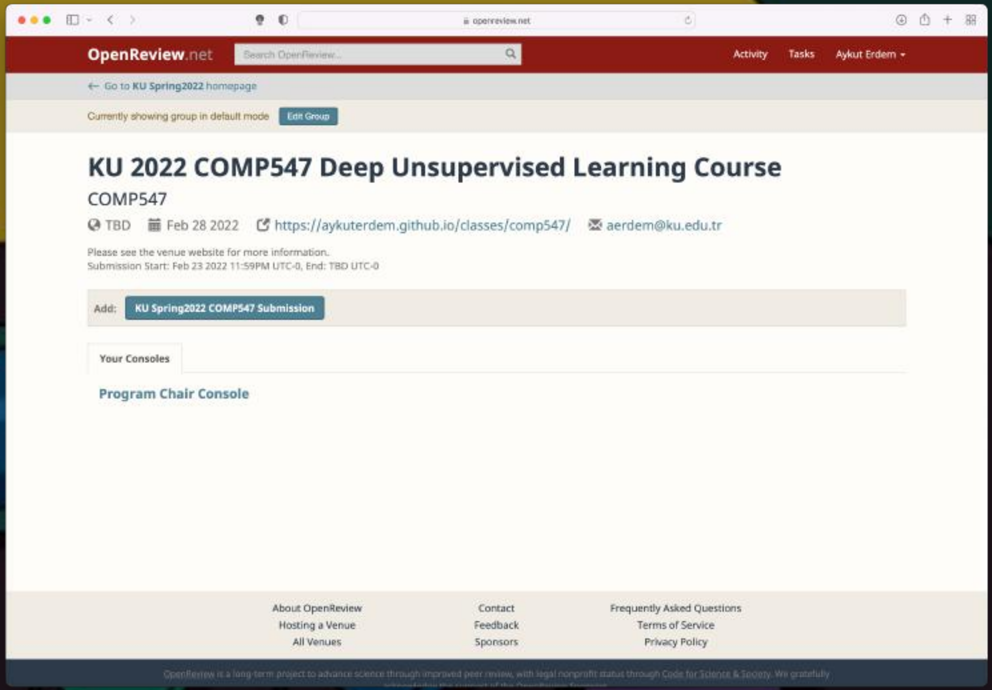
KOÇ UNIVERSITY

Aykut Erdem // Koç University // Spring 2022

Good news, everyone!

- Paper presentations start next week, on Wednesday (March 9)
- Each student should write a review on OpenReview

Good news, everyone!



- For paper reviews, we will use the OpenReview.net submission system.

Paper Reviews

Think deeply about the papers we read and try to learn from them as much as possible (and then even more). If you do not understand something, we should discuss it and dissect it together. Whatever you think others understand, they understand less (the instructor included), but together we will get it.

- Identify the key questions the paper studies, and the answers it provides to these questions.

- Consider the challenges of the problem or scenario studied, and how the paper's approach addresses them.

- Deconstruct the formal and technical parts to understand their fine details. Note to yourself aspects that are not clear to you

Borrowed from Yoav Artzi

Paper Reviewing Guidelines

- When reviewing the paper, start with 1–2 sentences summarizing what the paper is about.
- Continue with the strength of the paper. Outline its contribution, and your main takeaways. What did you learn?
- Highlight shortcomings and limitations. Please focus on weaknesses that fundamental to the method. Unlike conference or journal reviewing, this part is intended for your understanding and discussion.
- Try to suggest ways to address the paper's limitations. Any idea is welcome and will contribute to the discussion.
- Suggest questions for discussion in class. As part of the discussion in class, you are asked to raise these questions during the class.

Borrowed from Yoav Artzi

Previously on COMP547

- content-based attention
- location-based attention
- soft vs. hard attention
- case study: Show, Attend and Tell
- self-attention
- case study: Transformer networks

Lecture overview

- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- Autoregressive Models
  - Recurrent Neural Nets
  - Masking-based Models

Disclaimer: Much of the material and slides for this lecture were borrowed from
—Pieter Abbeel, Peter Chen, Jonathan Ho, Aravind Srinivas' Berkeley CS294-158 class

Lecture overview

- **Motivation**
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- Autoregressive Models
  - Recurrent Neural Nets
  - Masking-based Models

Likelihood-based models

- **Problems we'd like to solve:**
  - Generating data: synthesizing images, videos, speech, text
  - Compressing data: constructing efficient codes
  - Anomaly detection

- Likelihood-based models
  - Estimate p_{data} from samples $x^{(1)}, \ldots, x^{(n)} \sim p_{data}(x)$

- Learns a distribution p that allows:
  - Computing p(x) for arbitrary x
  - Sampling $x \sim p(x)$

- Today: **discrete** data

Desiderata

- We want to estimate distributions of **complex, high-dimensional data**
  - A 128×128×3 image lies in a ~50,000-dimensional space
- We also want computational and statistical efficiency
  - Efficient training and model representation
  - Expressiveness and generalization
  - Sampling quality and speed
  - Compression rate and speed

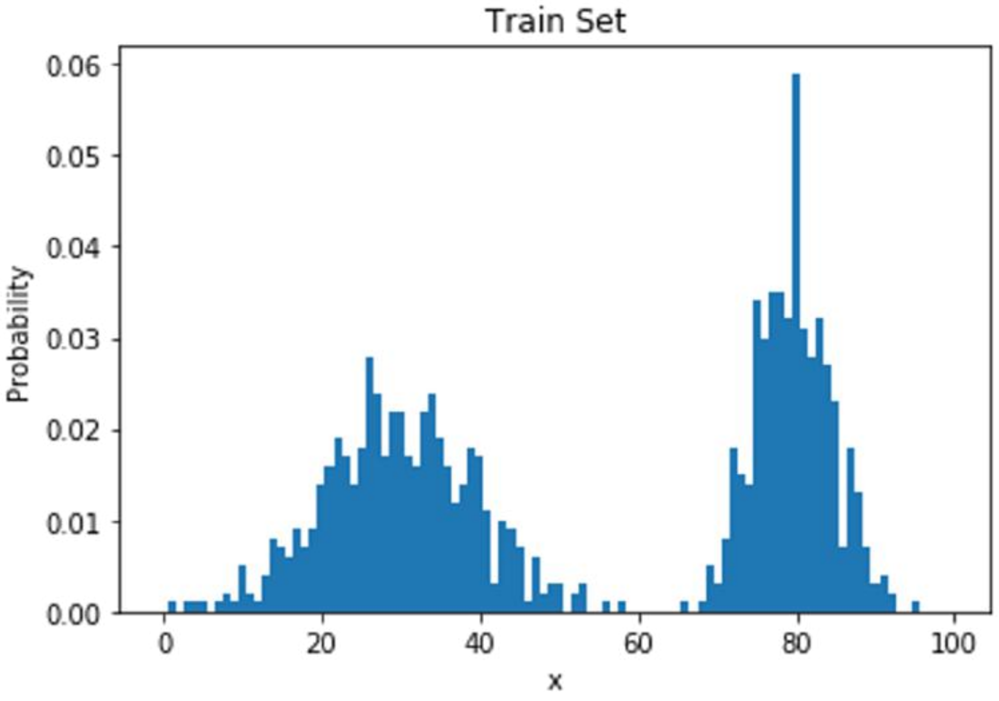
Lecture overview

- Motivation
- **Simple generative models: histograms**
- Parameterized distributions and maximum likelihood
- Autoregressive Models
  - Recurrent Neural Nets
  - Masking-based Models

Learning: Estimate frequencies by counting

- Recall: the goal is to estimate $\mathbf{p_{data}}$ **from samples $\mathbf{x^{(1)}, \ldots, x^{(n)} \sim p_{data}(x)}$**
- **Suppose** the samples take on values in a finite set {1, …, k}
- The model: a **histogram**
  - (Redundantly) described by k nonnegative numbers: $p_1, \ldots, p_k$
  - To train this model: count frequencies

    p_i = (# times i appears in the dataset) / (# points in the dataset)



- **How can we compute likelihoods and sample from this model?**

Inference and Sampling

- **Inference** (querying p_i for arbitrary i): simply a lookup into the array $p_1, \ldots, p_k$

- **Sampling** (lookup into the inverse cumulative distribution function)
  1. From the model probabilities $p_1, \ldots, p_k$, compute the cumulative distribution

     $$F_i = p_1 + \cdots + p_i \quad \text{for all } i \in \{1, \ldots, k\}$$

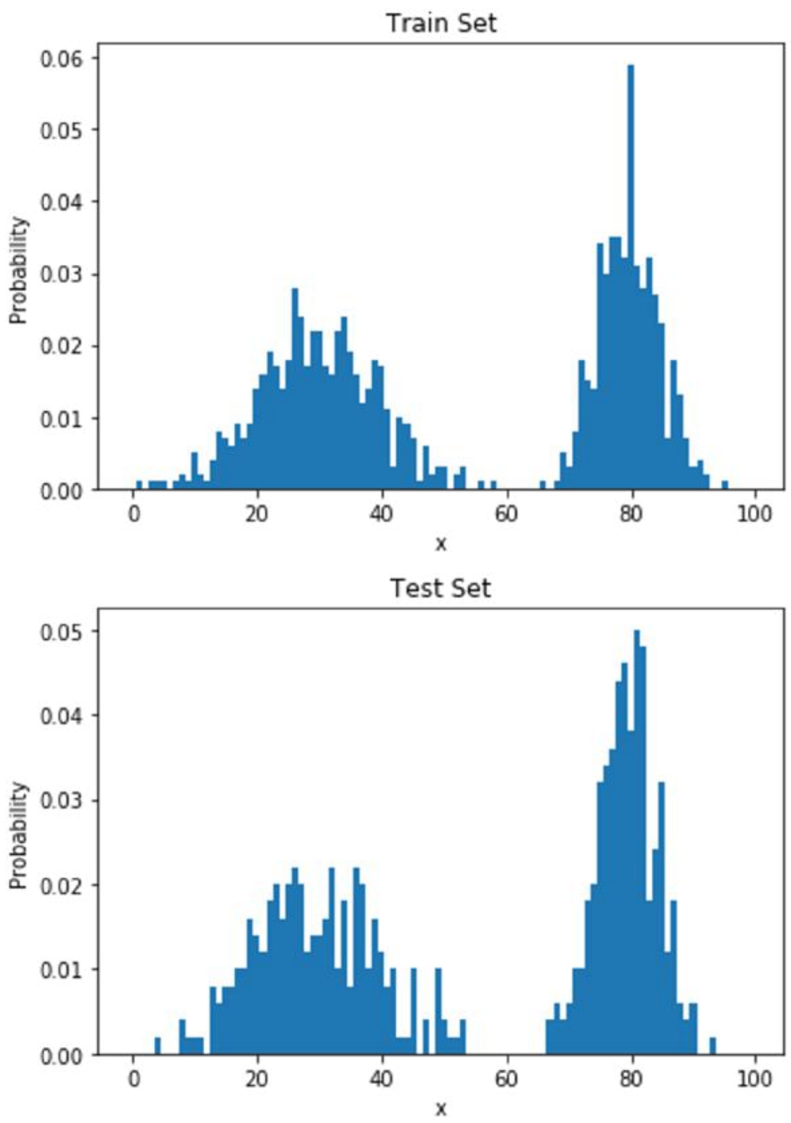
  2. Draw a uniform random number $u \sim [0, 1]$
  3. Return the smallest i such that $u \leq F_i$

- **Are we done?**

Failure in high dimensions

- No, because of the **curse of dimensionality**. Counting fails when there are too many bins.
  - (Binary) MNIST: 28x28 images, each pixel in {0, 1}
  - There are $2^{784} \approx 10^{236}$ probabilities to estimate
  - Any reasonable training set covers only a tiny fraction of this
  - No generalization whatsoever! There is shared structure among images but this kind of model would not be able to capture that.

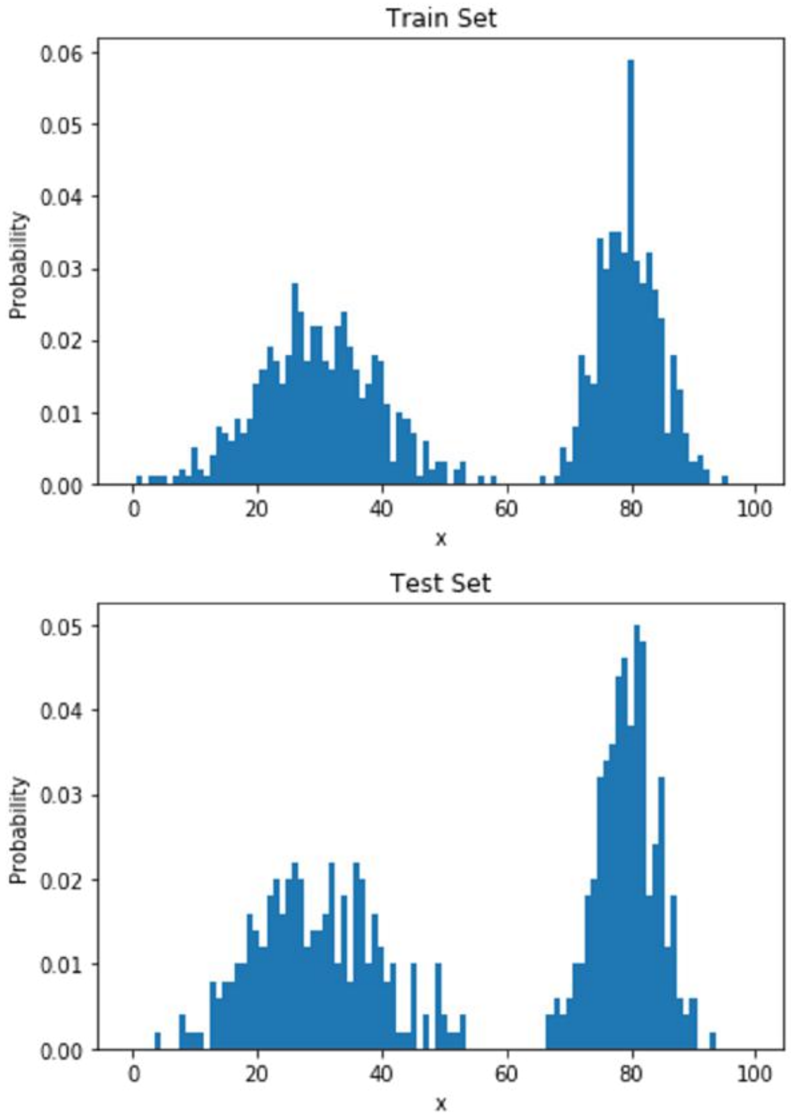
Problematic even for single variable

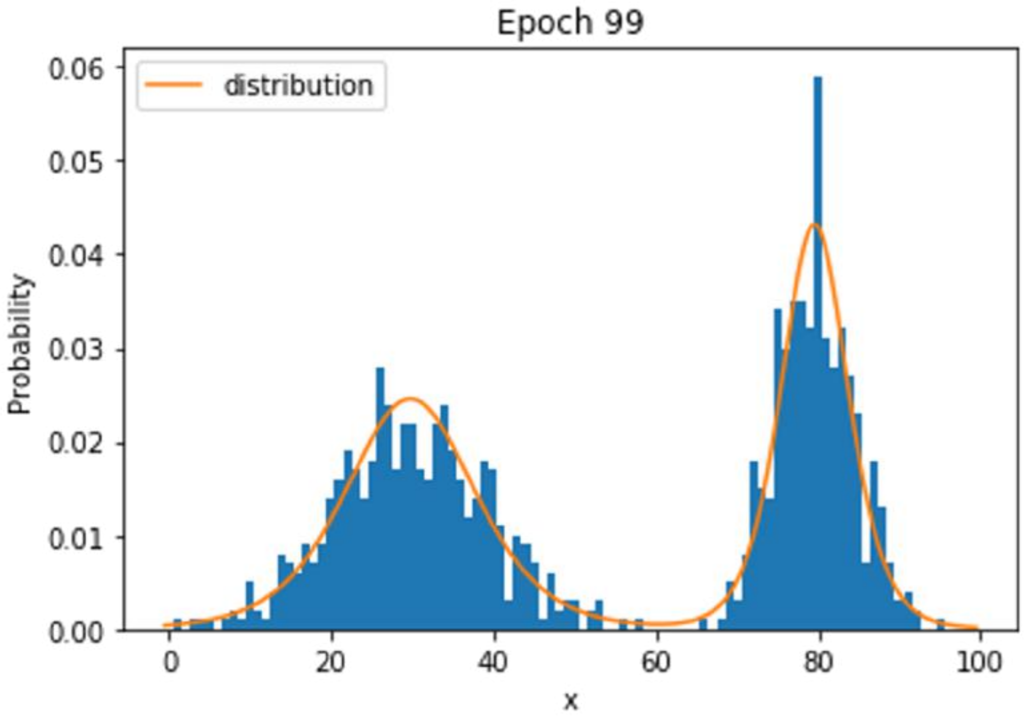


learned histogram = training data distribution

→ often poor generalization

Parameterized Distributions





Fitting a parameterized distribution often generalizes better

Lecture overview

- Motivation
- Simple generative models: histograms
- **Parameterized distributions and maximum likelihood**
- Autoregressive Models
  - Recurrent Neural Nets
  - Masking-based Models

Likelihood-based generative models

- Recall: the goal is to **estimate p_{data} from $x^{(1)}, \ldots, x^{(n)} \sim p_{data}(x)$**

- Now we introduce **function approximation**: learn θ so that $p_\theta(x) \approx p_{data}(x)$.
  - How do we design function approximators to effectively represent complex joint distributions over x, yet remain easy to train?
  - There will be many choices for model design, each with different tradeoffs and different compatibility criteria.

- **Designing the model and the training procedure go hand-in-hand.**

Fitting distributions

- Given data $\mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}$ sampled from a "true" distribution p_{data}
- Set up a model class: a set of parameterized distributions p_θ
- Pose a search problem over parameters

$$\arg\min_\theta \ \mathrm{loss}(\theta, \mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)})$$

- Want the loss function + search procedure to:
  - Work with large datasets (n is large, say millions of training examples)
  - Yield θ such that p_θ matches p_{data} — i.e. the training algorithm works. Think of the loss as a distance between distributions.
  - Note that the training procedure can only see the empirical data distribution, not the true data distribution: we want the model to generalize.

Maximum likelihood

- Maximum likelihood: given a dataset $x^{(1)}$, ..., $x^{(n)}$, find θ by solving the optimization problem

$$\arg\min_{\theta} \ \text{loss}(\theta, \mathbf{x}^{(1)}, \ldots, \mathbf{x}^{(n)}) = \frac{1}{n}\sum_{i=1}^{n} -\log p_{\theta}(\mathbf{x}^{(i)})$$

- Statistics tells us that if the model family is expressive enough and if enough data is given, then solving the maximum likelihood problem will yield parameters that generate the data
- Equivalent to minimizing KL divergence between the empirical data distribution and the model

$$\hat{p}_{\text{data}}(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}[\mathbf{x} = \mathbf{x}^{(i)}]$$

$$\text{KL}(\hat{p}_{\text{data}} \,\|\, p_{\theta}) = \mathbb{E}_{\mathbf{x}\sim\hat{p}_{\text{data}}}[-\log p_{\theta}(\mathbf{x})] - H(\hat{p}_{\text{data}})$$

Stochastic gradient descent

- Maximum likelihood is an optimization problem. How do we solve it?
- **Stochastic gradient descent** (SGD).
  - SGD minimizes expectations: for f a differentiable function of θ, it solves

$$\arg\min_{\theta} \mathbb{E}[f(\theta)]$$

  - With maximum likelihood, the optimization problem is

$$\arg\min_{\theta} \mathbb{E}_{\mathbf{x}\sim\hat{p}_{\text{data}}}[-\log p_{\theta}(\mathbf{x})]$$

  - **Why maximum likelihood + SGD?** It works with large datasets and is compatible with neural networks.

Designing the model

- **Key requirement for maximum likelihood + SGD: efficiently compute log p(x) and its gradient**
- **We will choose models p_θ to be deep neural networks**, which work in the regime of high expressiveness and efficient computation (assuming specialized hardware)
- **How exactly do we design these networks?**
  - Any setting of θ must define a valid probability distribution over x:

$$\text{for all } \theta, \quad \sum_{\mathbf{x}} p_\theta(\mathbf{x}) = 1 \quad \text{and} \quad p_\theta(\mathbf{x}) \geq 0 \quad \text{for all } \mathbf{x}$$

  - log p_θ(x) should be easy to evaluate and differentiate with respect to θ
  - This can be tricky to set up!

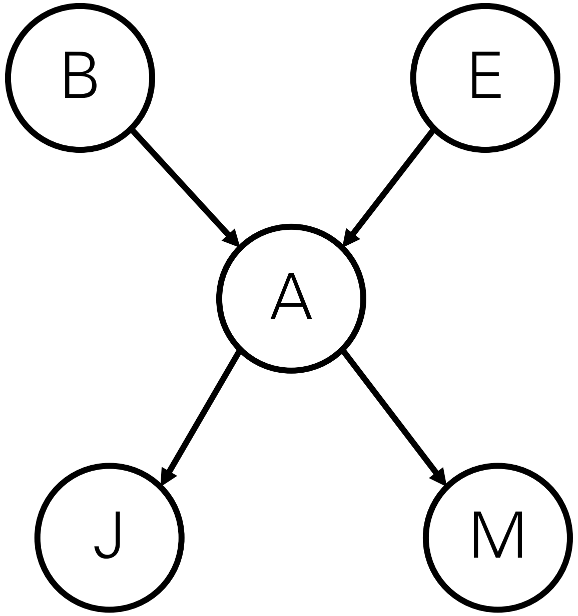
Bayes nets and neural nets

- **Main idea:** place a **Bayes net** structure (a directed acyclic graph) over the variables in the data, and model the conditional distributions with neural networks.

- **Reduces the problem to designing conditional likelihood-based models for single variables**. We know how to do this: the neural net takes variables being conditioned on as input, and outputs the distribution for the variable being predicted.

B	P(B)
+b	0.001
-b	0.999

E	P(E)
+e	0.002
-e	0.998



A	J	P(J|A)
+a	+j	0.9
+a	-j	0.1
-a	+j	0.05
-a	-j	0.95

A	M	P(M|A)
+a	+m	0.7
+a	-m	0.3
-a	+m	0.01
-a	-m	0.99

$$P(B,E,A,J,M) = P(B)\underbrace{P(E|B)}_{P(E)}P(A|E,B)\underbrace{P(J|A,E,B)}_{P(J|A)}\underbrace{P(M|J,A,E,B)}_{P(M|A)}$$ chain rule

Lecture overview

- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- **Autoregressive Models**
  – Recurrent Neural Nets
  – Masking-based Models

Autoregressive models

- First, given a Bayes net structure, setting the conditional distributions to neural networks will yield a tractable log likelihood and gradient. Great for maximum likelihood training!

$$\log p_\theta(\mathbf{x}) = \sum_{i=1}^{d} \log p_\theta(x_i \mid \text{parents}(x_i))$$

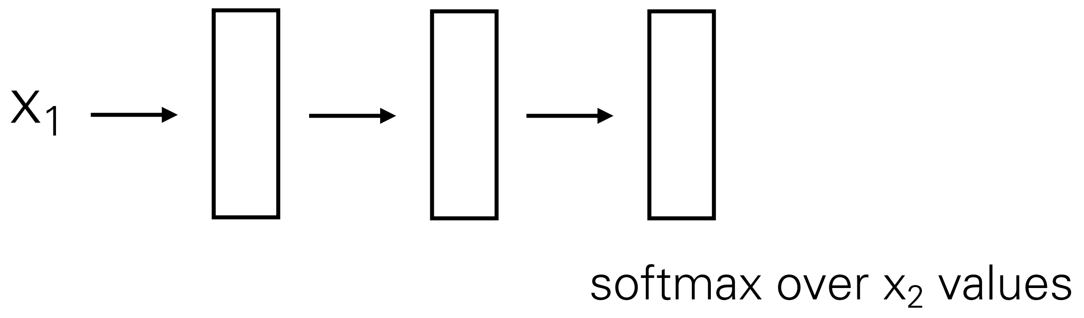
- But is it expressive enough? Yes, assuming a fully expressive Bayes net structure: any joint distribution can be written as a product of conditionals

$$\log p(\mathbf{x}) = \sum_{i=1}^{d} \log p(x_i \mid \mathbf{x}_{1:i-1})$$

- This is called an **autoregressive model**. So, an expressive Bayes net structure with neural network conditional distributions yields an expressive model for p(x) with tractable maximum likelihood training.

A toy autoregressive model

- Two variables: x_1, x_2
- Model: $p(x_1, x_2) = p(x_1)\, p(x_2|x_1)$
  - $p(x_1)$ is a histogram
  - $p(x_2|x_1)$ is a multilayer perceptron
    - Input is x_1
    - Output is a distribution over x_2 (logits, followed by softmax)



One function approximator per conditional

- Does this extend to high dimensions?
  - Somewhat. For d-dimensional data, O(d) parameters
    - Much better than O(exp(d)) in tabular case
    - What about text generation where d can be arbitrarily large?
  - Limited generalization
    - No information sharing among different conditionals
- Solution: share parameters among conditional distributions. Two approaches:
  - Recurrent neural networks
  - Masking

Lecture overview

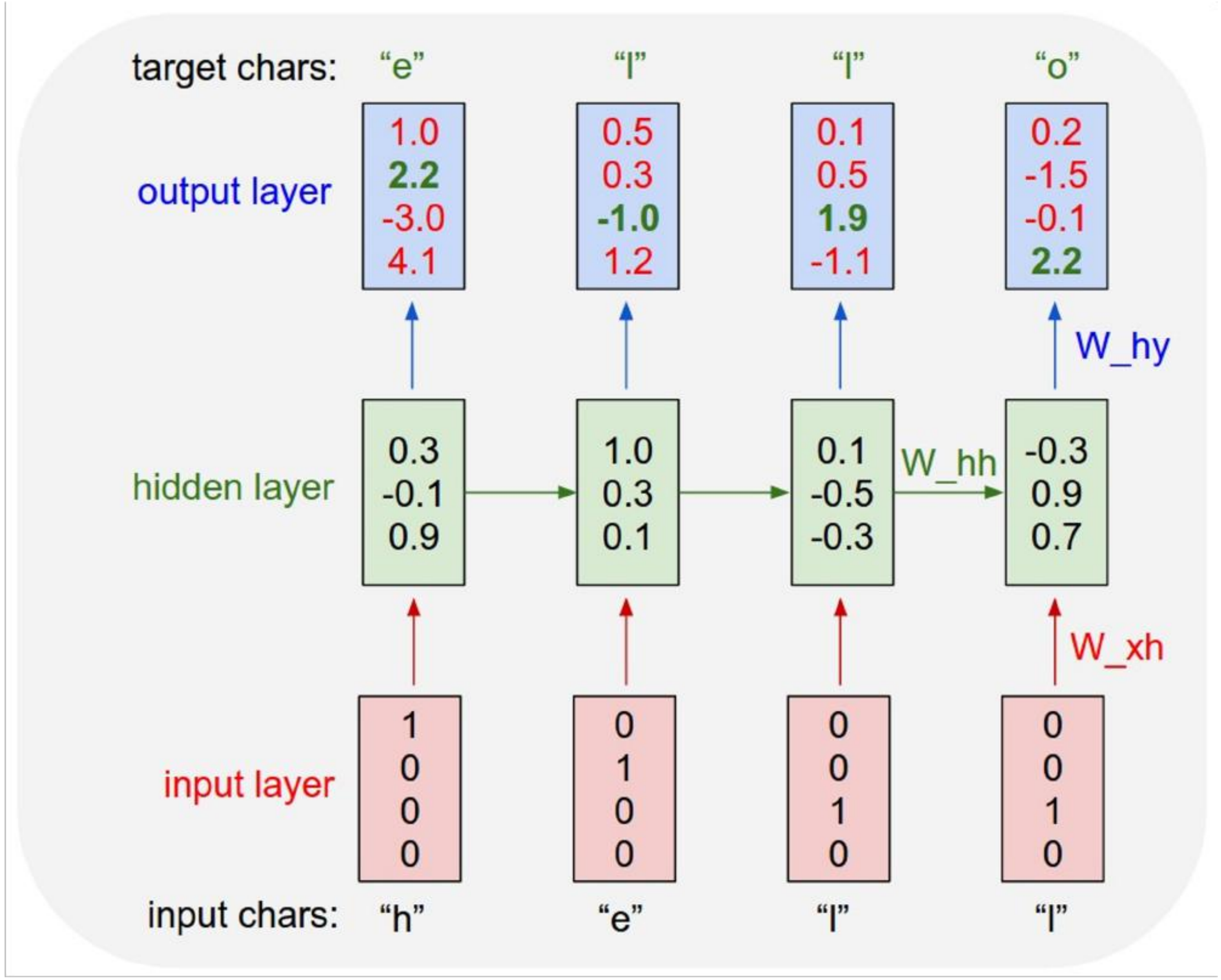
- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- **Autoregressive Models**
  - **Recurrent Neural Nets**
  - Masking-based Models

RNN autoregressive models - char-rnn

$$\log p(\mathbf{x}) = \sum_{i=1}^{d} \log p(x_i \mid \mathbf{x}_{1:i-1})$$

Sequence of characters

Character at i^{th} position



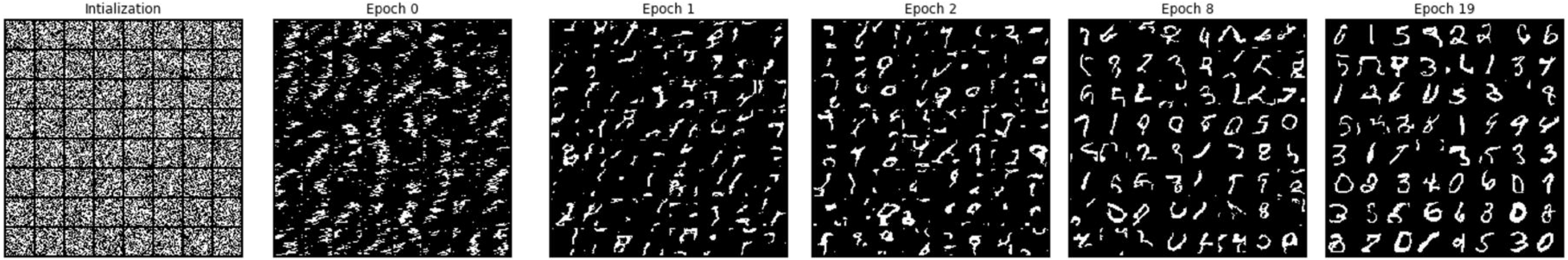
[Karpathy, 2015]

MNIST

- Handwritten digits
- 28x28
- 60,000 train
- 10,000 test

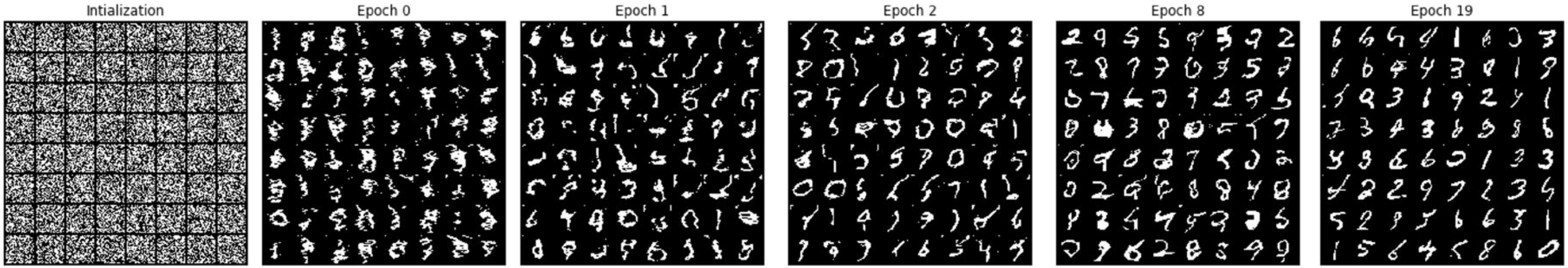
- Original: greyscale
- “Binarized MNIST” -- 0/1 (black/white)

RNN on MNIST



RNN with Pixel Location Appended on MNIST

- Append (x,y) coordinates of pixel in the image as input to RNN



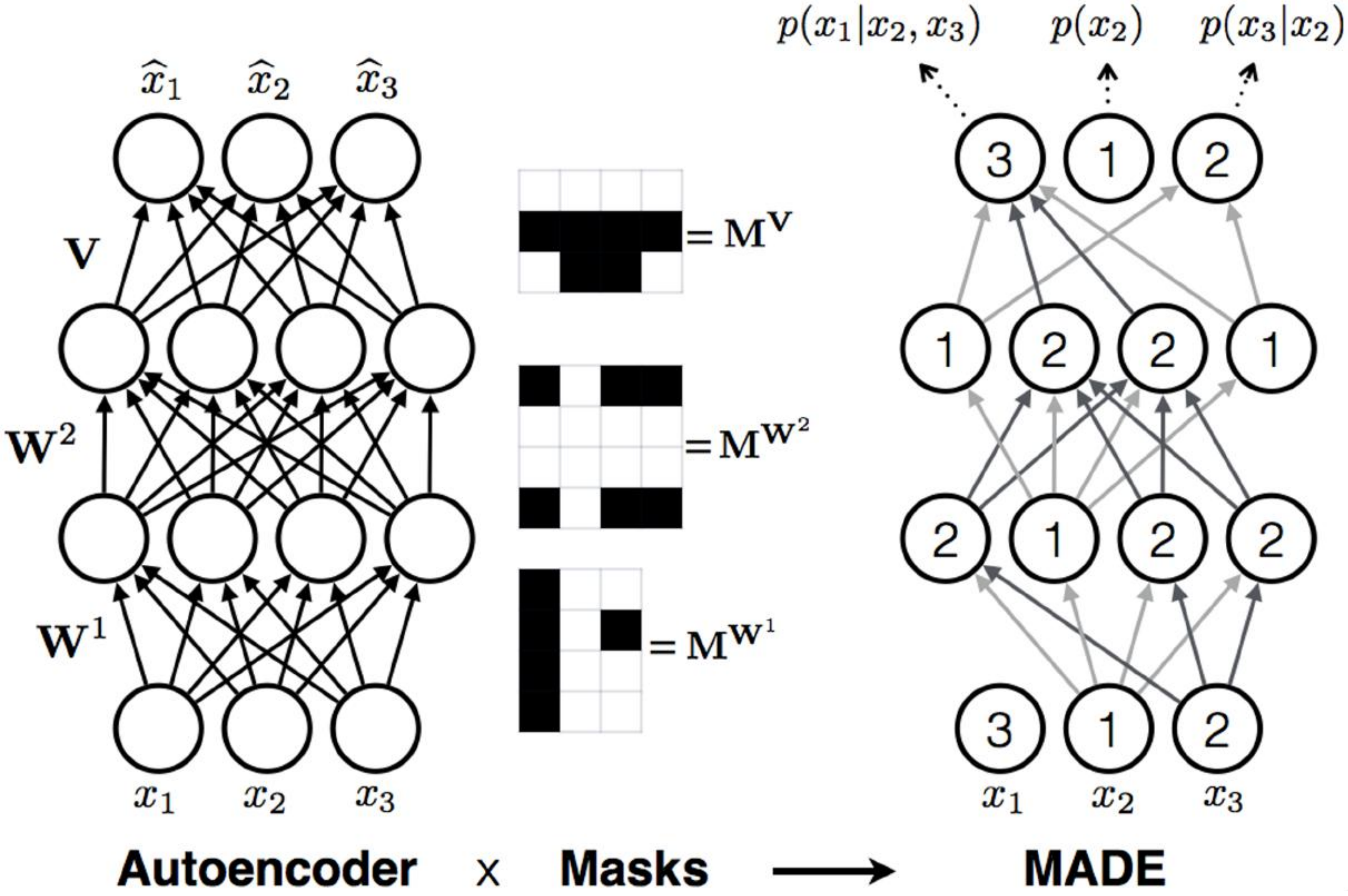
Lecture overview

- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- **Autoregressive Models**
  - Recurrent Neural Nets
  - **Masking-based Models**
    - **MADE**
    - Masked Convolutions
    - WaveNet
    - PixelCNN (+ variations)

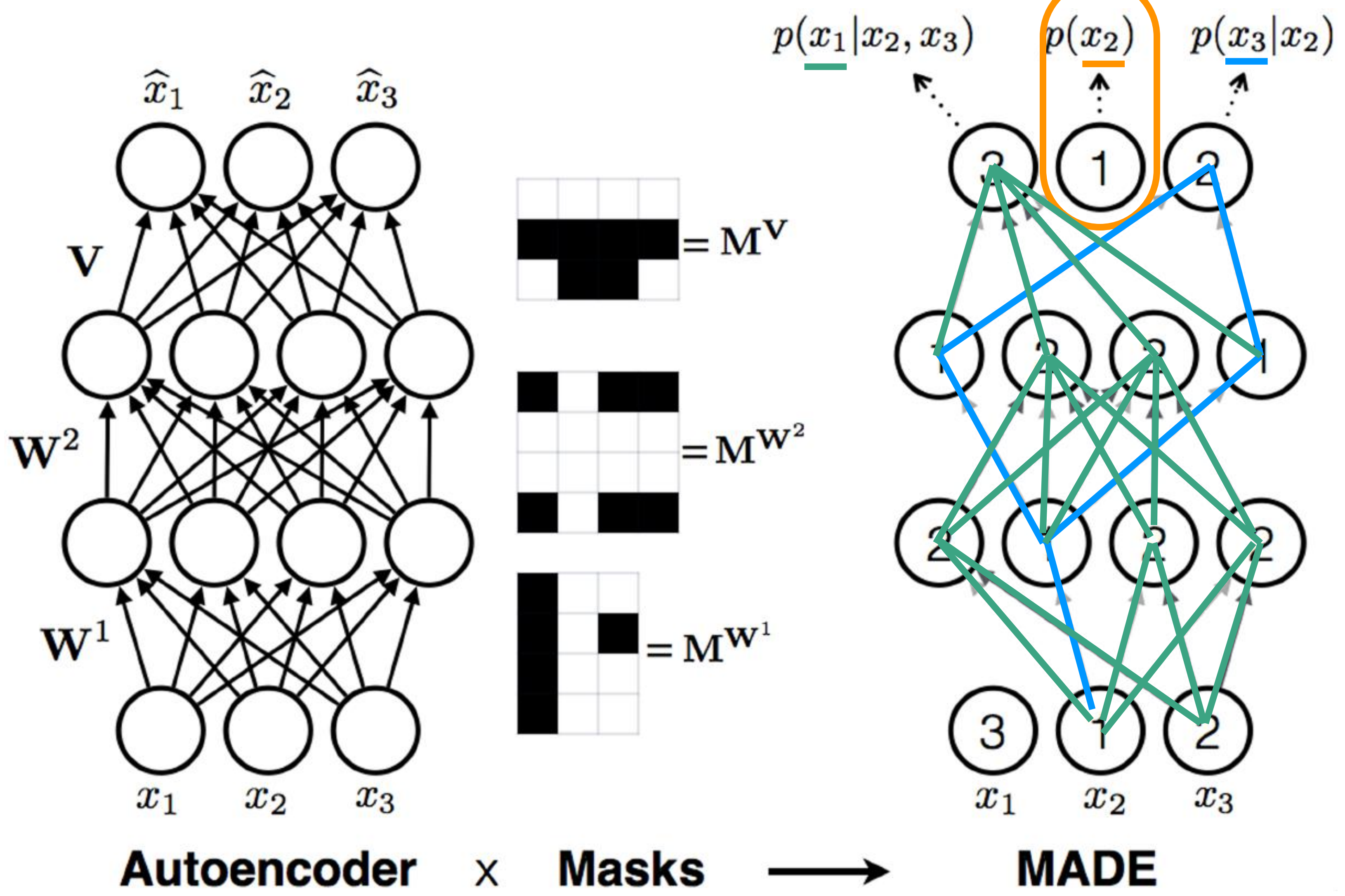
Masking-based autoregressive models

- Second major branch of neural AR models
  - Key property: parallelized computation of all conditionals
  - Masked MLP (MADE)
  - Masked convolutions & self-attention
    - Also share parameters across time

Masked Autoencoder for Distribution Estimation (MADE)



Masked Autoencoder for Distribution Estimation (MADE)



$p(x_2)$

$p(x_3|x_2)$

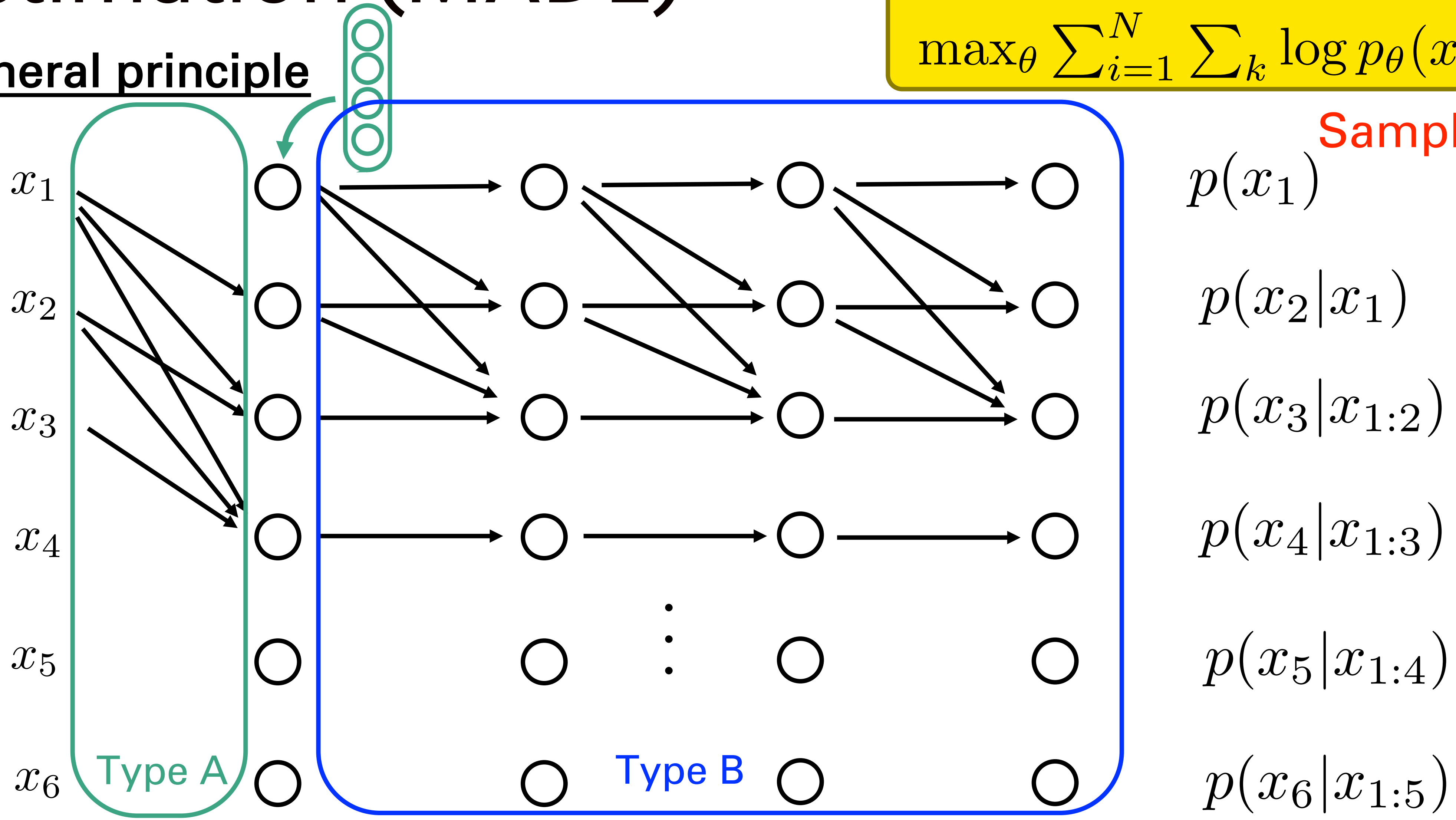
$p(x_1|x_2, x_3)$

Masked Autoencoder for Distribution Estimation (MADE)

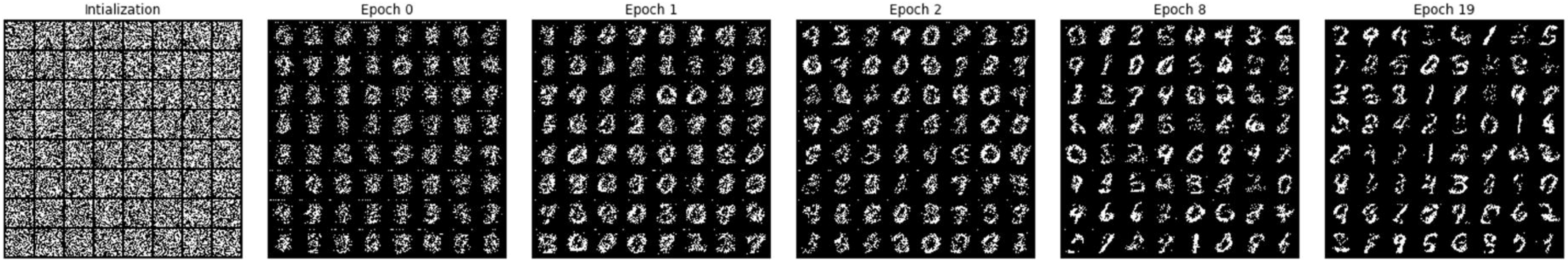
$$p(x) = p(x_1) \cdot p(x_2|x_1) \cdots p(x_6|x_{1:5})$$
$$\max_\theta \sum_{i=1}^N \sum_k \log p_\theta(x_k^{(i)}|x_{1:k-1}^{(i)})$$

Sampling is slow!

<u>General principle</u>



MADE on MNIST



MADE results

Table 6. Negative log-likelihood test results of different models on the binarized MNIST dataset.

Model	$-\log p$	
RBM (500 h, 25 CD steps)	≈ 86.34	Intractable
DBM 2hl	≈ 84.62	
DBN 2hl	≈ 84.55	
DARN n_h=500	≈ 84.71	
DARN n_h=500, adaNoise	≈ 84.13	
MoBernoullis K=10	168.95	Tractable
MoBernoullis K=500	137.64	
NADE 1hl (fixed order)	88.33	
EoNADE 1hl (128 orderings)	87.71	
EoNADE 2hl (128 orderings)	85.10	
MADE 1hl (1 mask)	88.40	
MADE 2hl (1 mask)	89.59	
MADE 1hl (32 masks)	88.04	
MADE 2hl (32 masks)	86.64	

MADE results

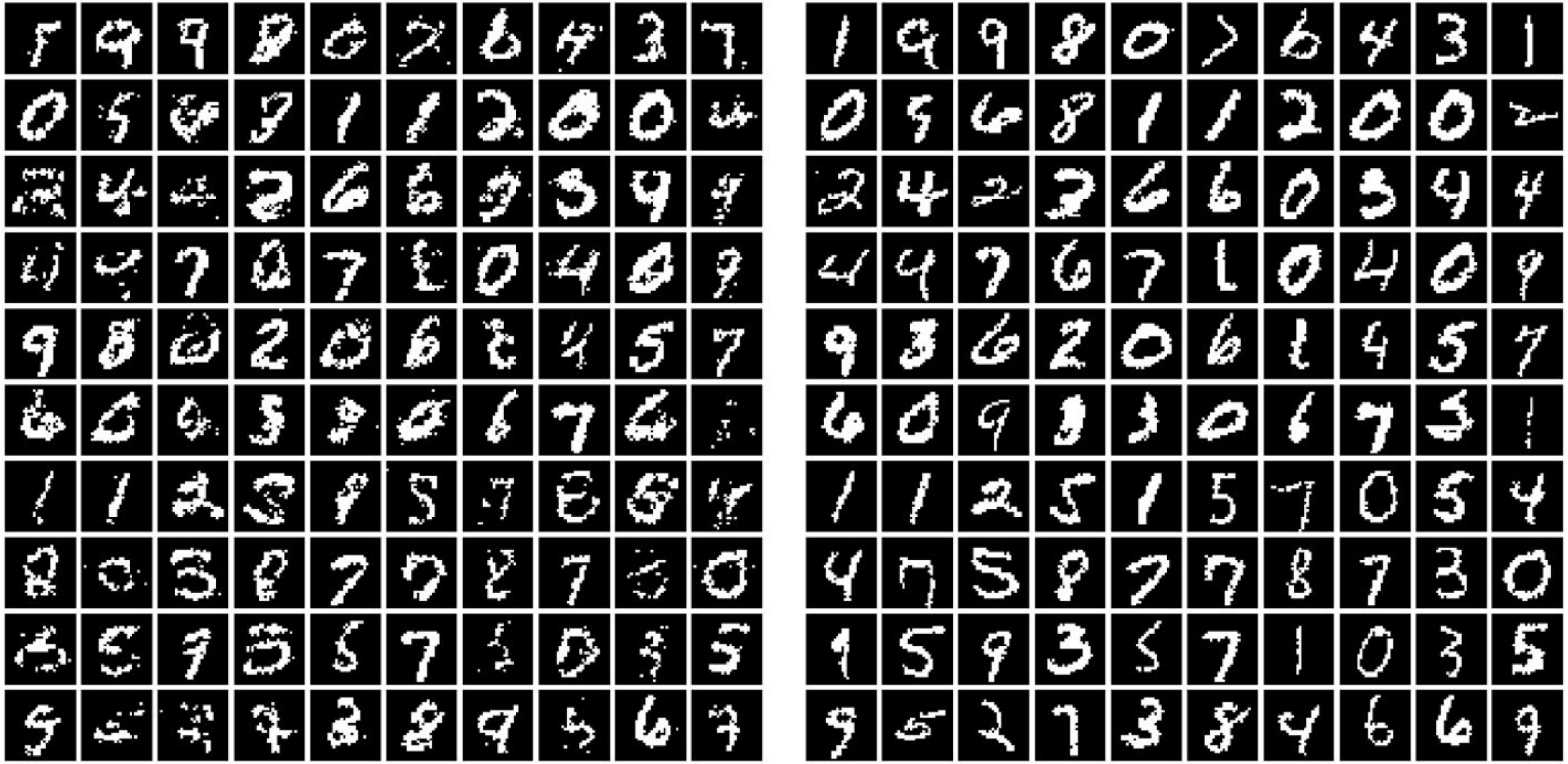
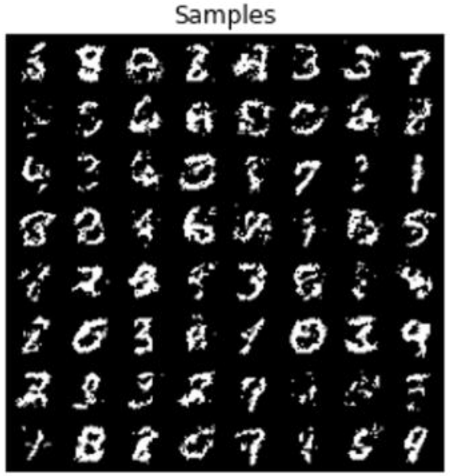


Figure 3. Left: Samples from a 2 hidden layer MADE. Right: Nearest neighbour in binarized MNIST.

MADE -- Different Orderings

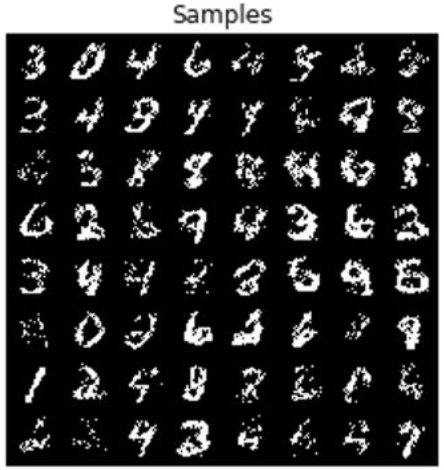
- All orderings achieve roughly the same bits per dim $\left(\frac{\log_2 p(x)}{\dim(x)}\right)$, but samples are different

Random Permutation

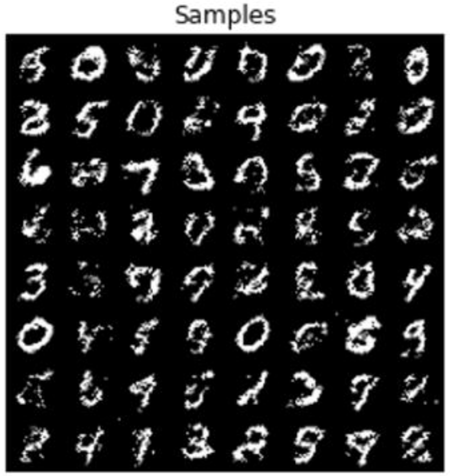


Even then Odd Indices

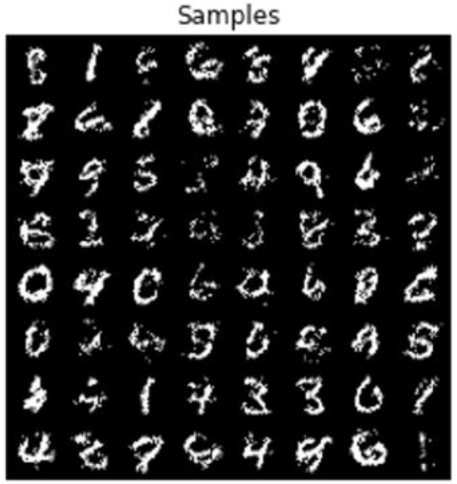
Rows (Raster Scan)



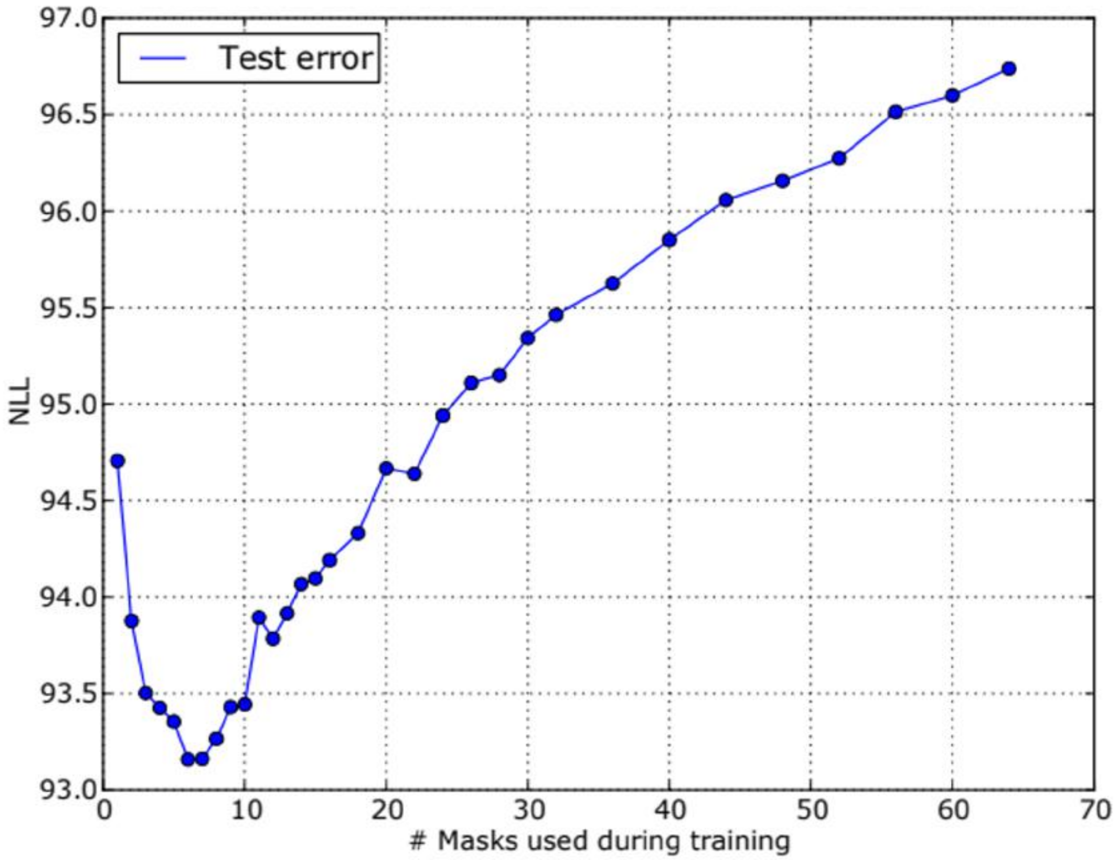
Columns



Top to Middle, Bottom to Middle



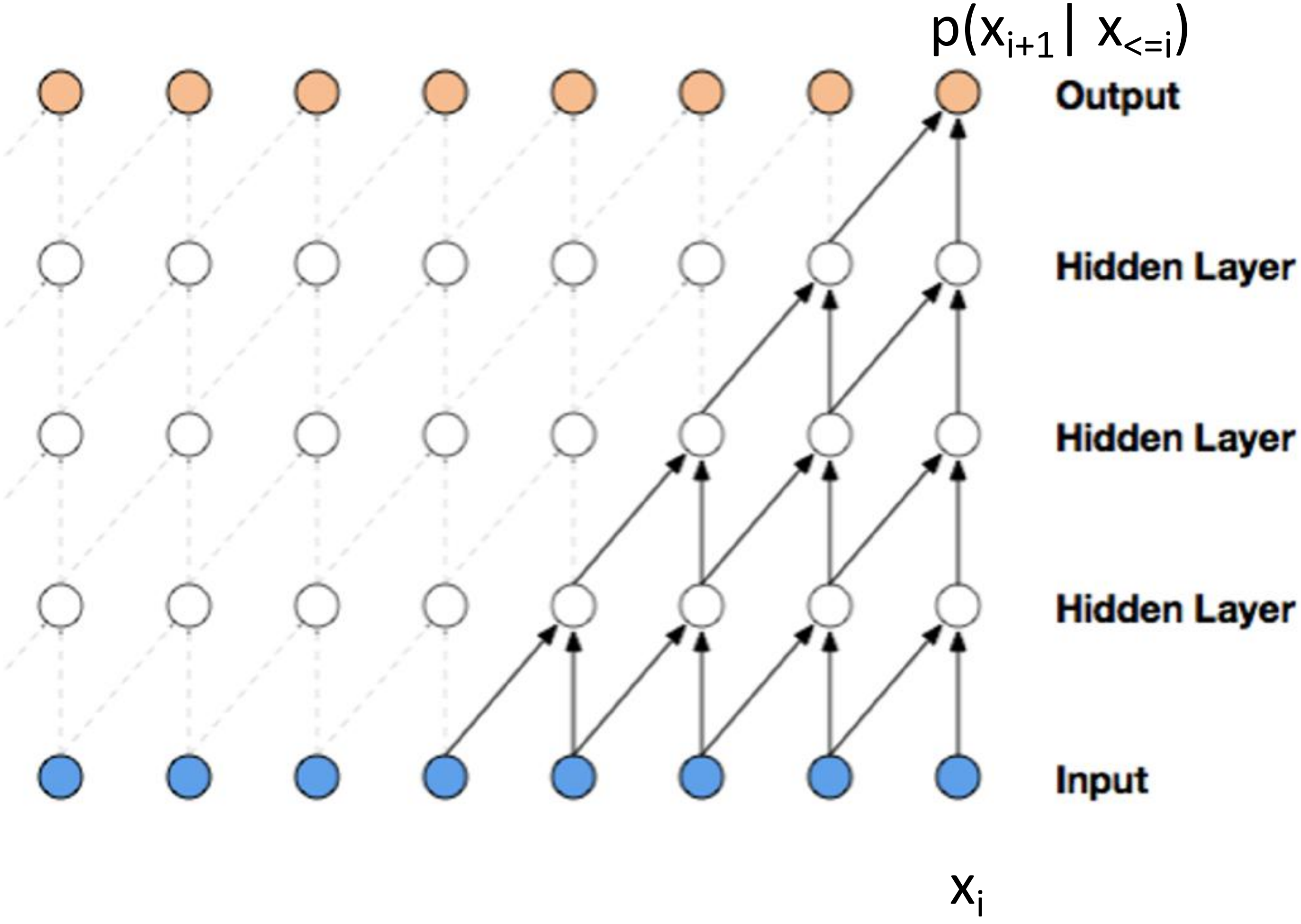
MADE: Multiple Orderings



Lecture overview

- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- **Autoregressive Models**
  - Recurrent Neural Nets
  - **Masking-based Models**
    - MADE
    - **Masked Convolutions**
      - **WaveNet**
      - PixelCNN (+ variations)

Masked Temporal (1D) Convolution



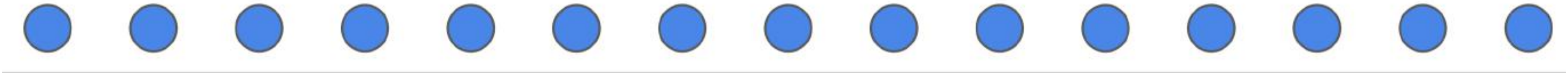
- Easy to implement, masking part of the conv kernel
- Constant parameter count for variable-length distribution!
- Efficient to compute, convolution has hyper-optimized implementations on all hardware

However

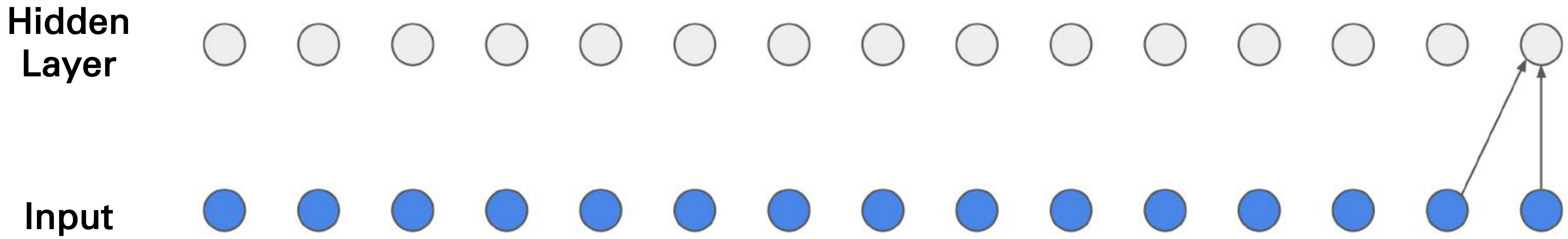
- Limited receptive field, linear in number of layers

WaveNet – Causal Dilated Convolution

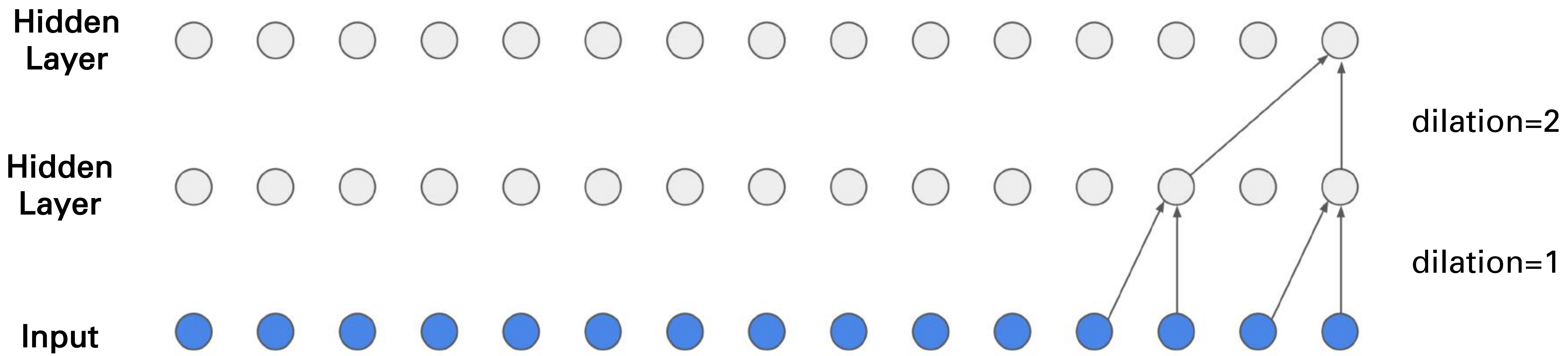
Input



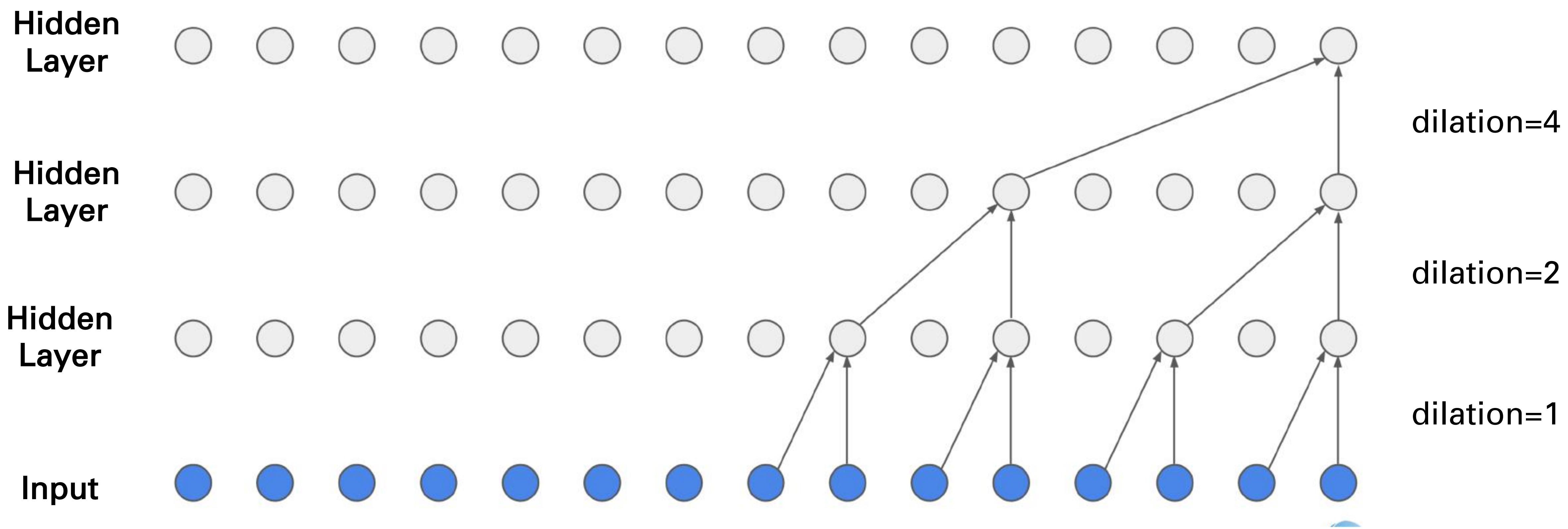
WaveNet – Causal Dilated Convolution



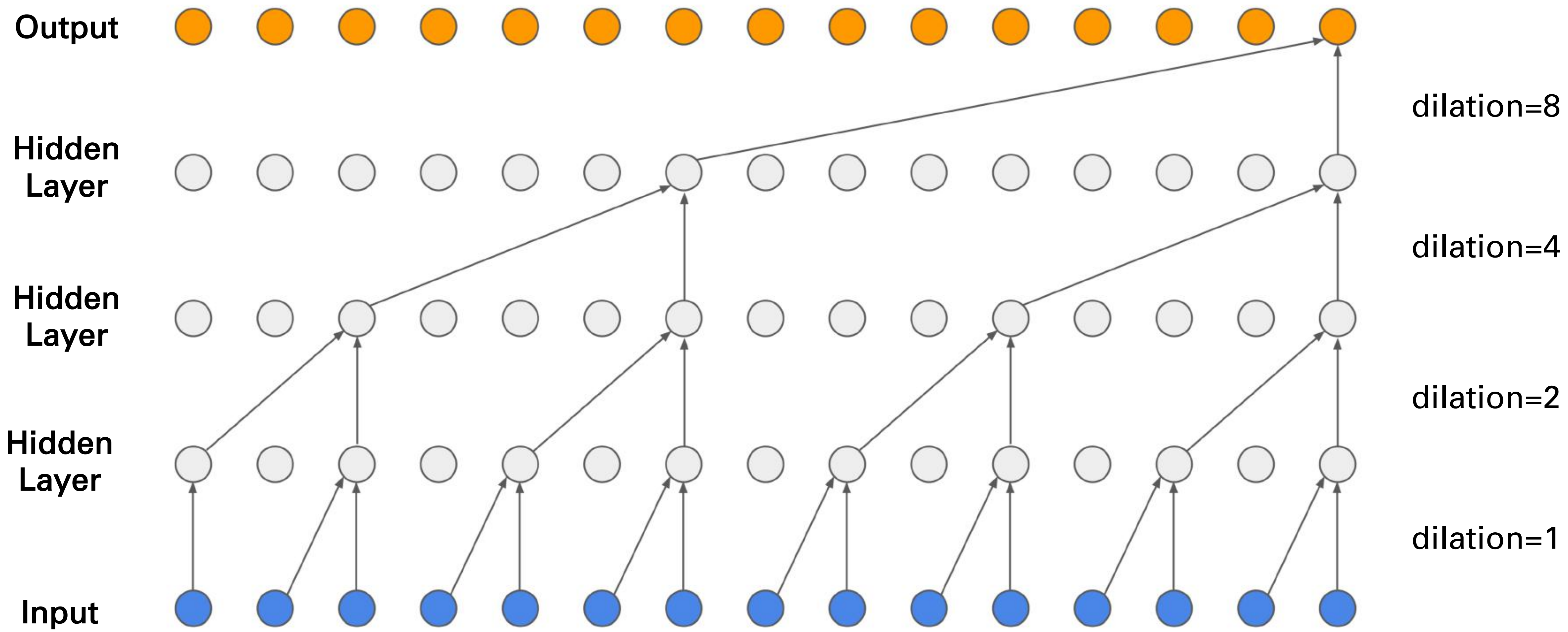
WaveNet – Causal Dilated Convolution



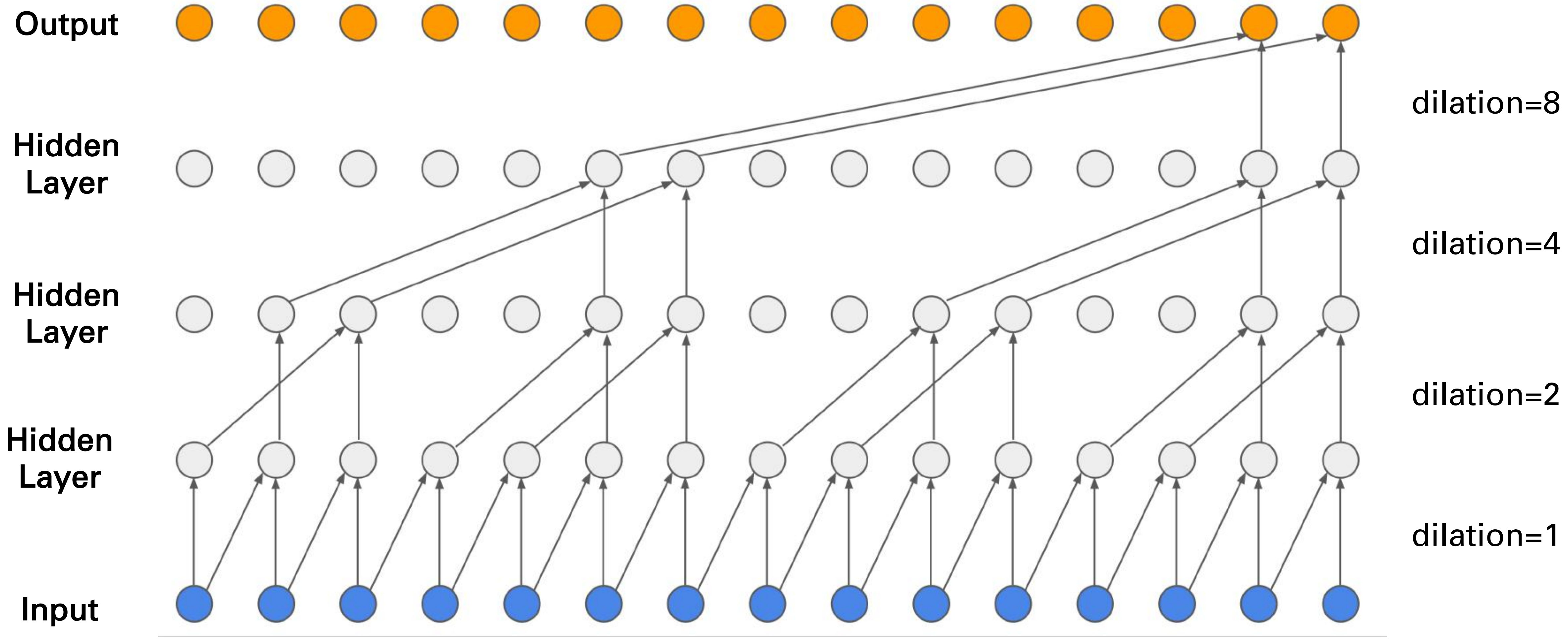
WaveNet – Causal Dilated Convolution



WaveNet – Causal Dilated Convolution

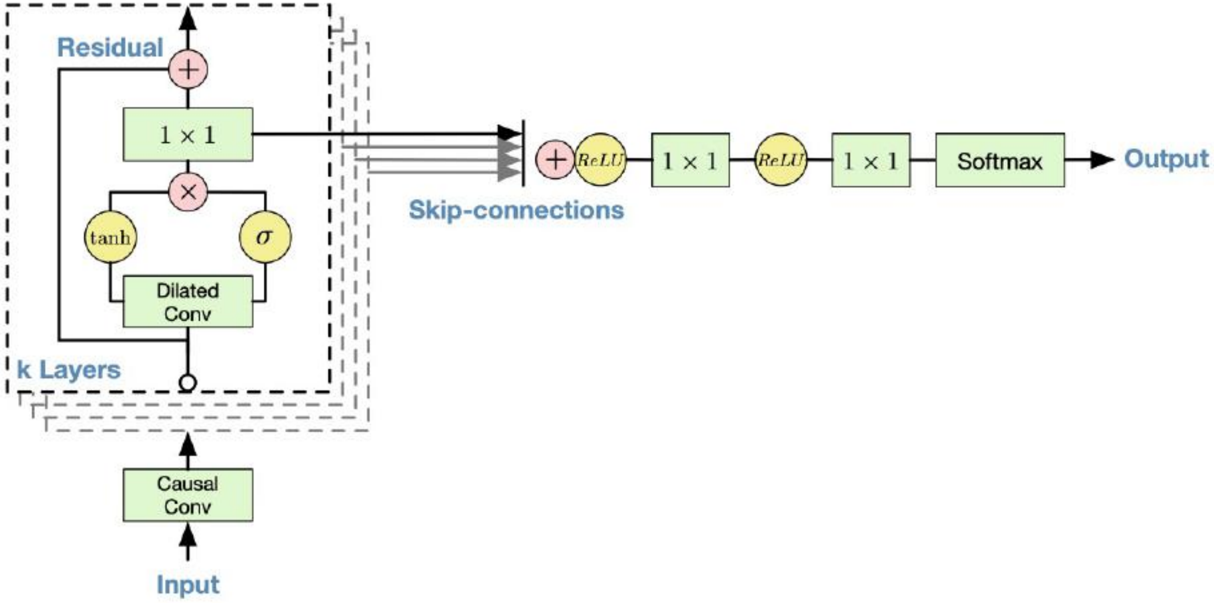


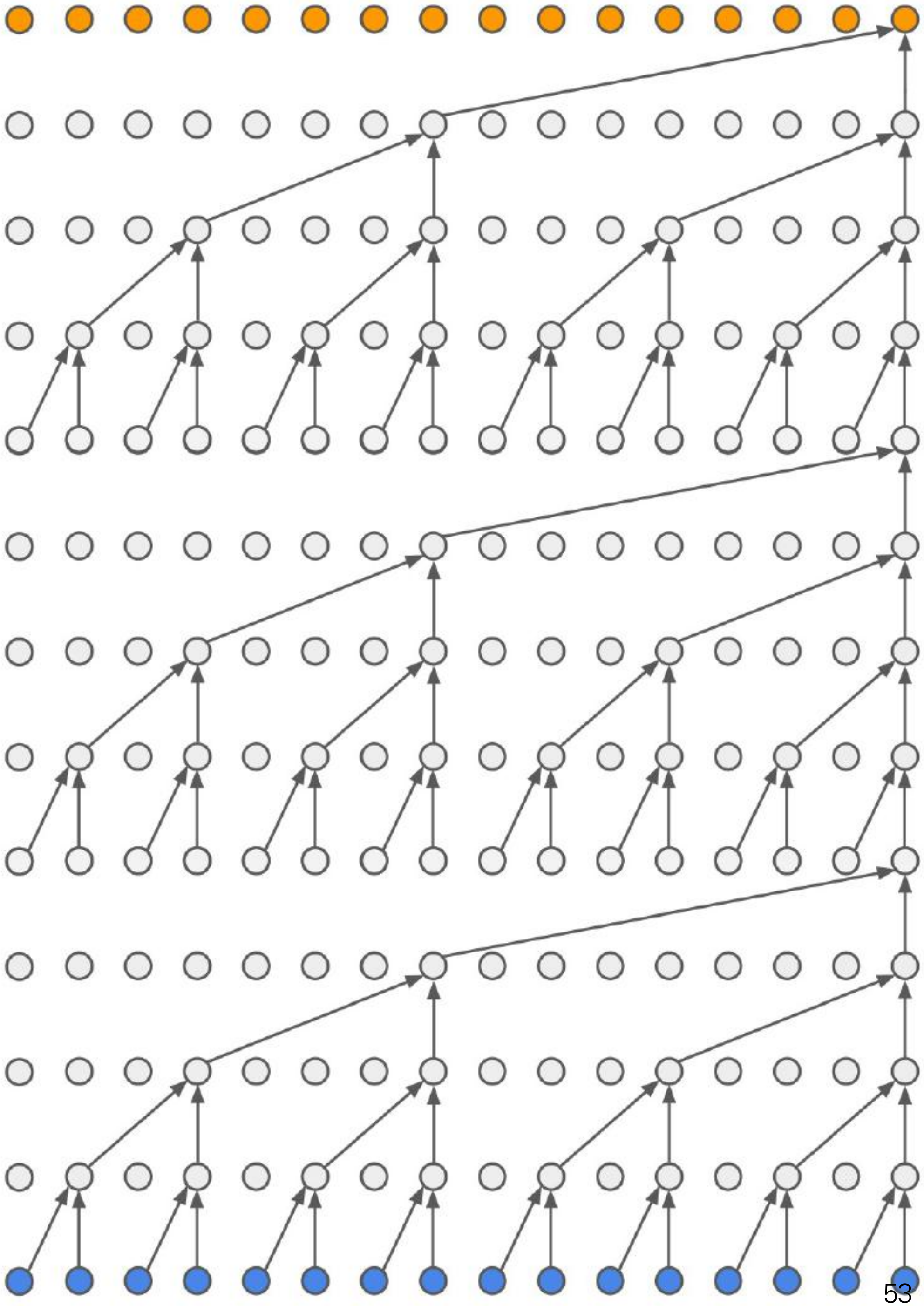
WaveNet – Causal Dilated Convolution



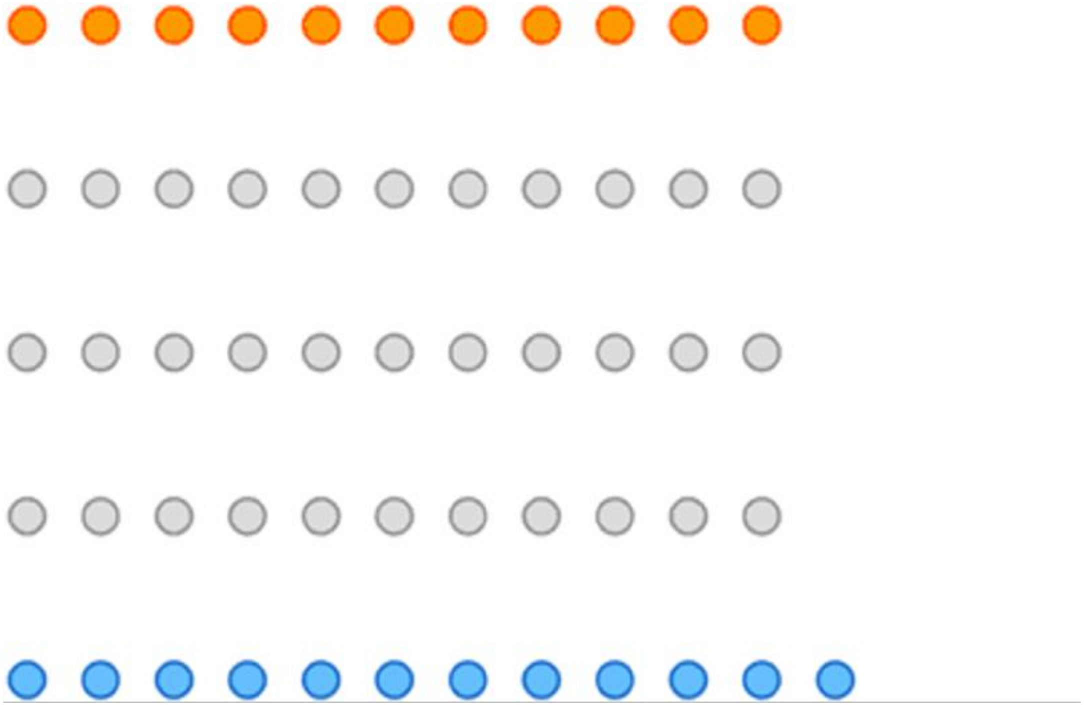
WaveNet – Multiple Stacks

- Improved receptive field with dilated convolutions
- Gated Residual block with skip connections



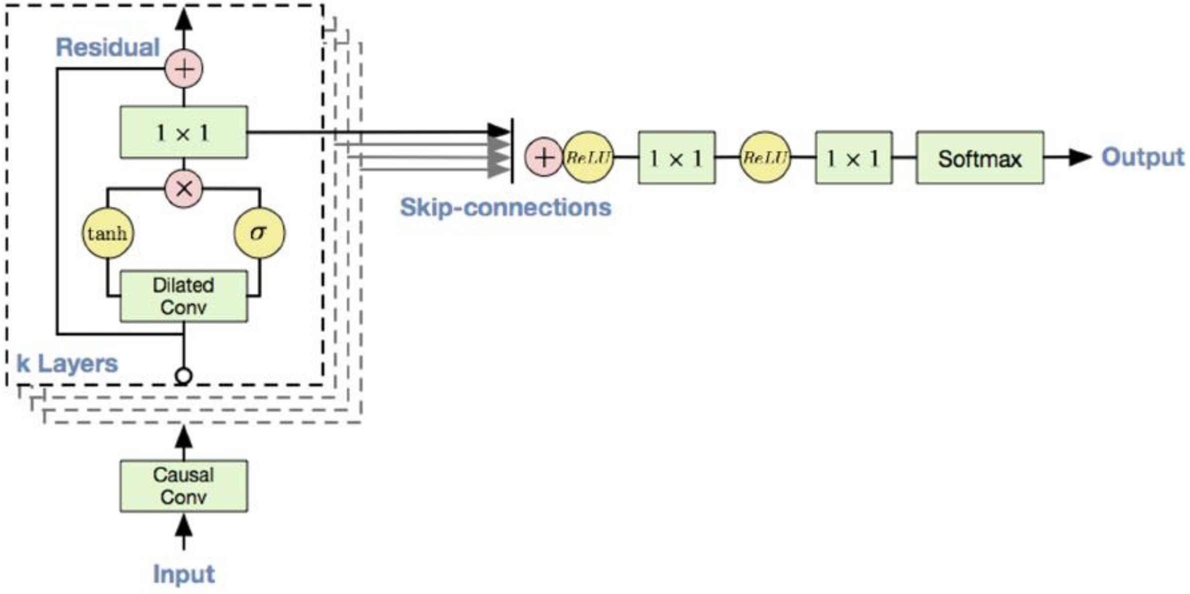


WaveNet

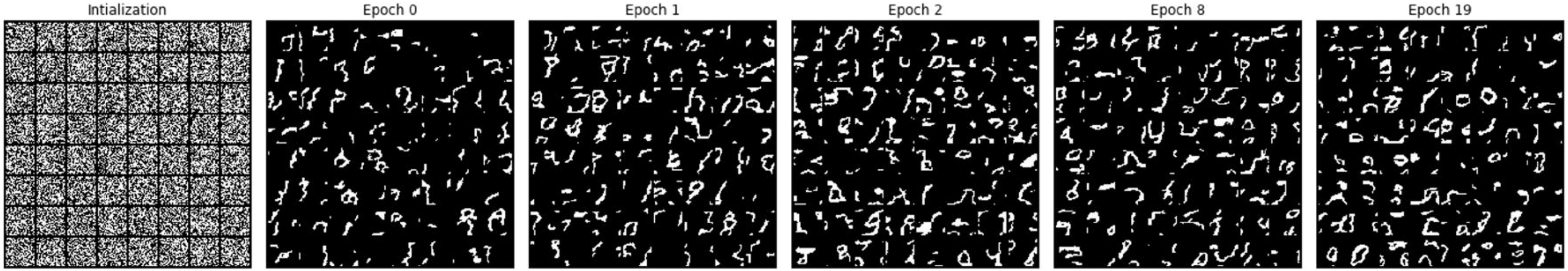


- Improved receptive field: dilated convolution, with exponential dilation

- Better expressivity: Gated Residual blocks, Skip connections

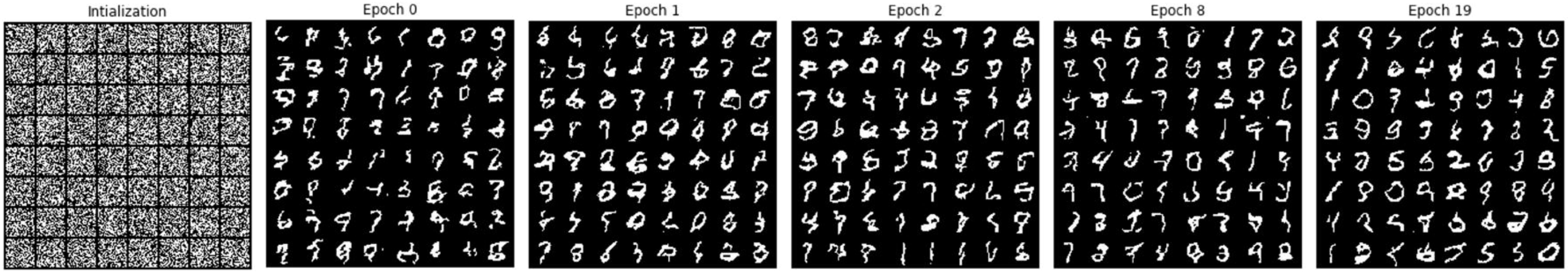


WaveNet on MNIST



WaveNet with Pixel Location Appended on MNIST

- Append (x,y) coordinates of pixel in the image as input to WaveNet

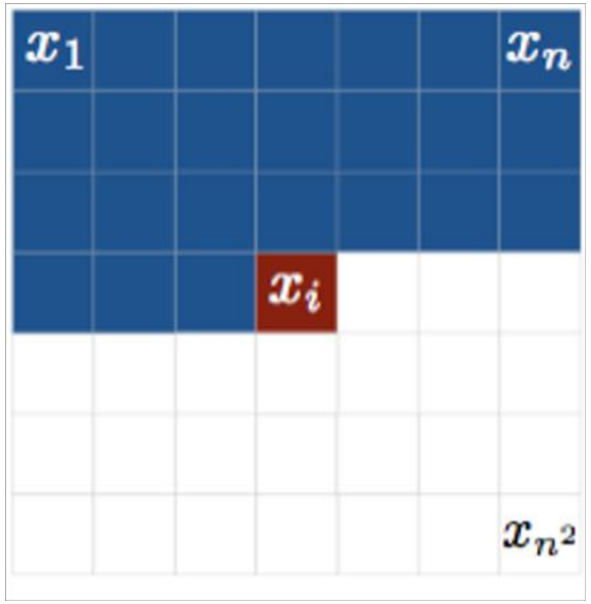


Lecture overview

- Motivation
- Simple generative models: histograms
- Parameterized distributions and maximum likelihood
- **Autoregressive Models**
  - Recurrent Neural Nets
  - **Masking-based Models**
    - MADE
    - **Masked Convolutions**
      - WaveNet
      - **PixelCNN (+ variations)**

Masked Spatial (2D) Convolution - PixelCNN

- Images can be flatten into 1D vectors, but they are fundamentally 2D
- We can use a masked variant of ConvNet to exploit this knowledge
- First, we impose an autoregressive ordering on 2D images:

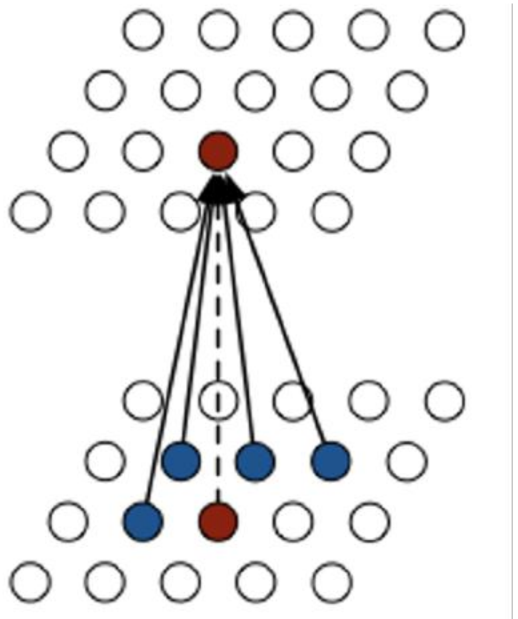


This is called raster scan ordering.
(Different orderings are possible,
more on this later)

PixelCNN

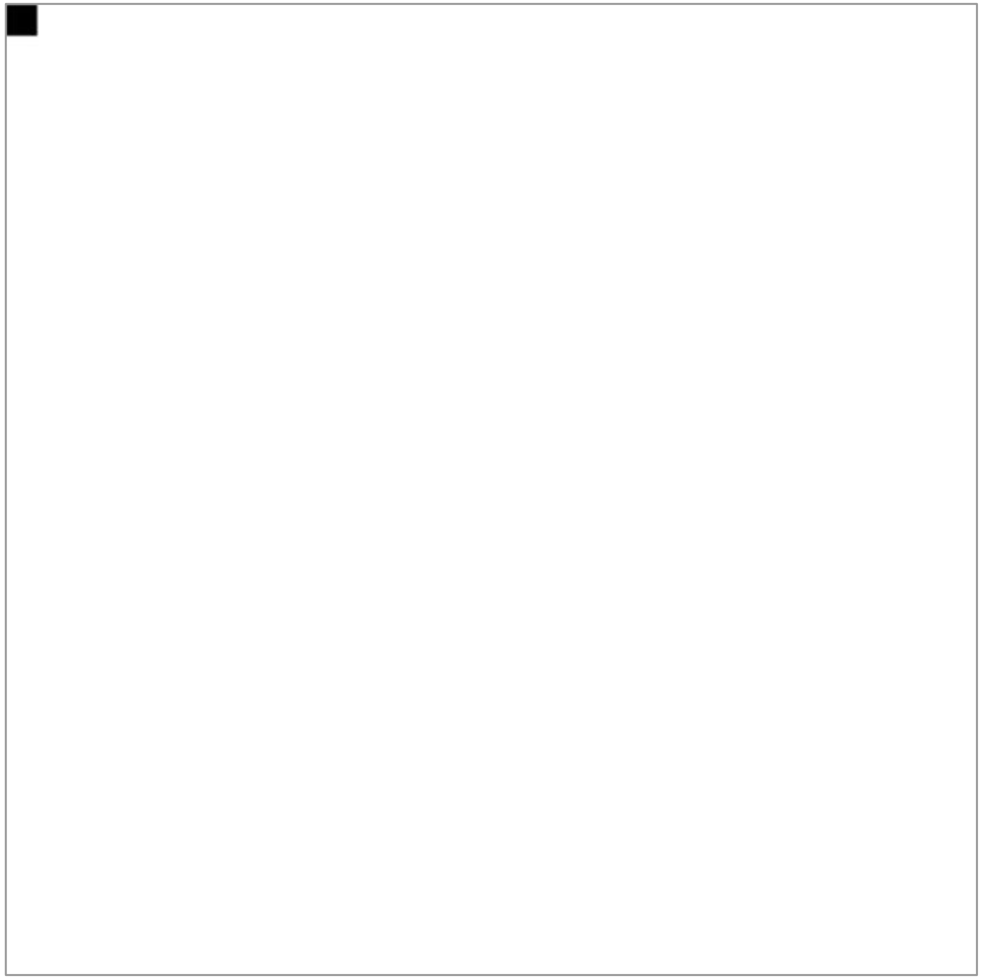
- Design question: how to design a masking method to obey that ordering?
- One possibility: PixelCNN (2016)

1	1	1
1	0	0
0	0	0

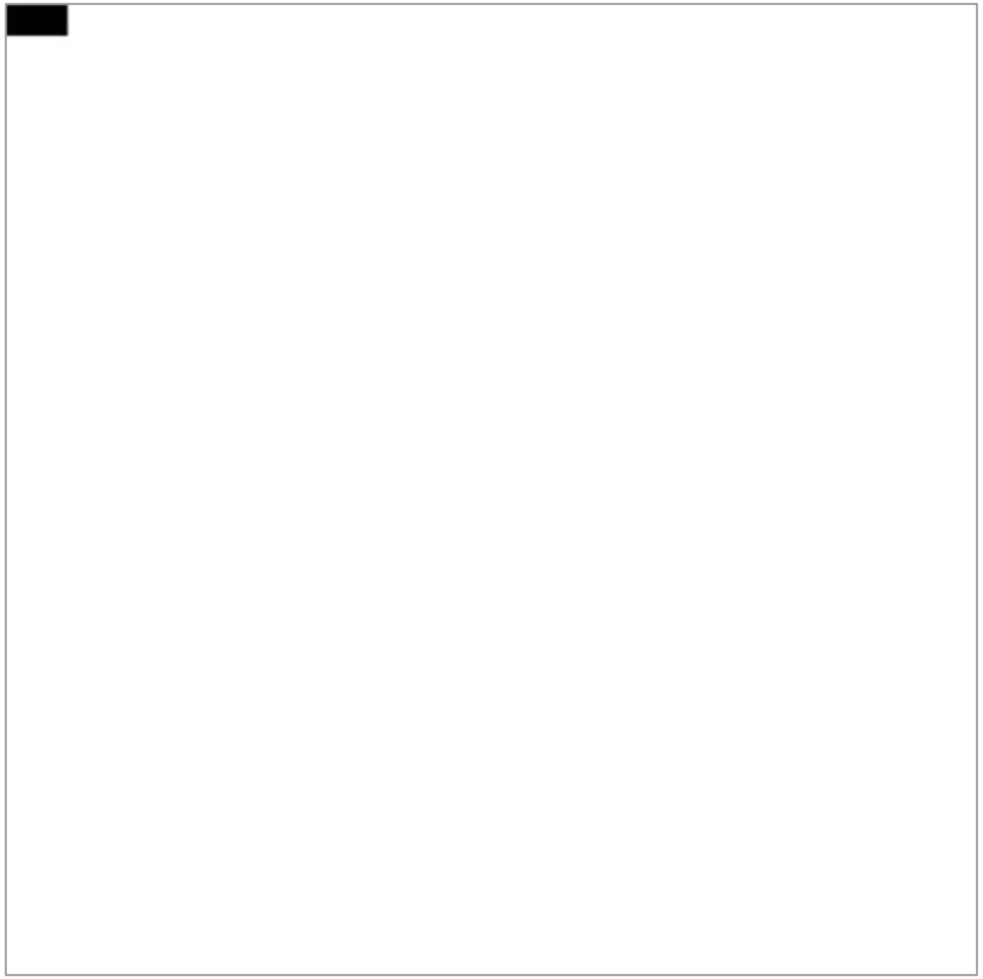


PixelCNN

Softmax Sampling

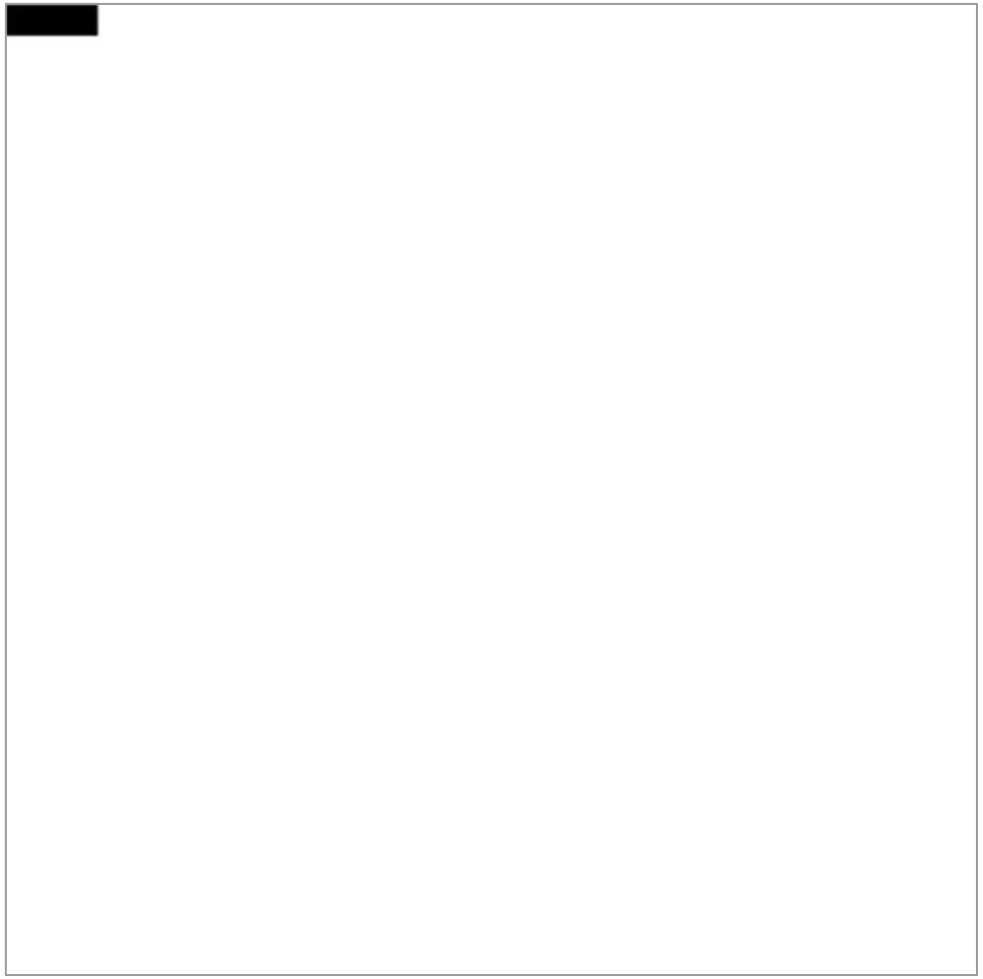


Softmax Sampling



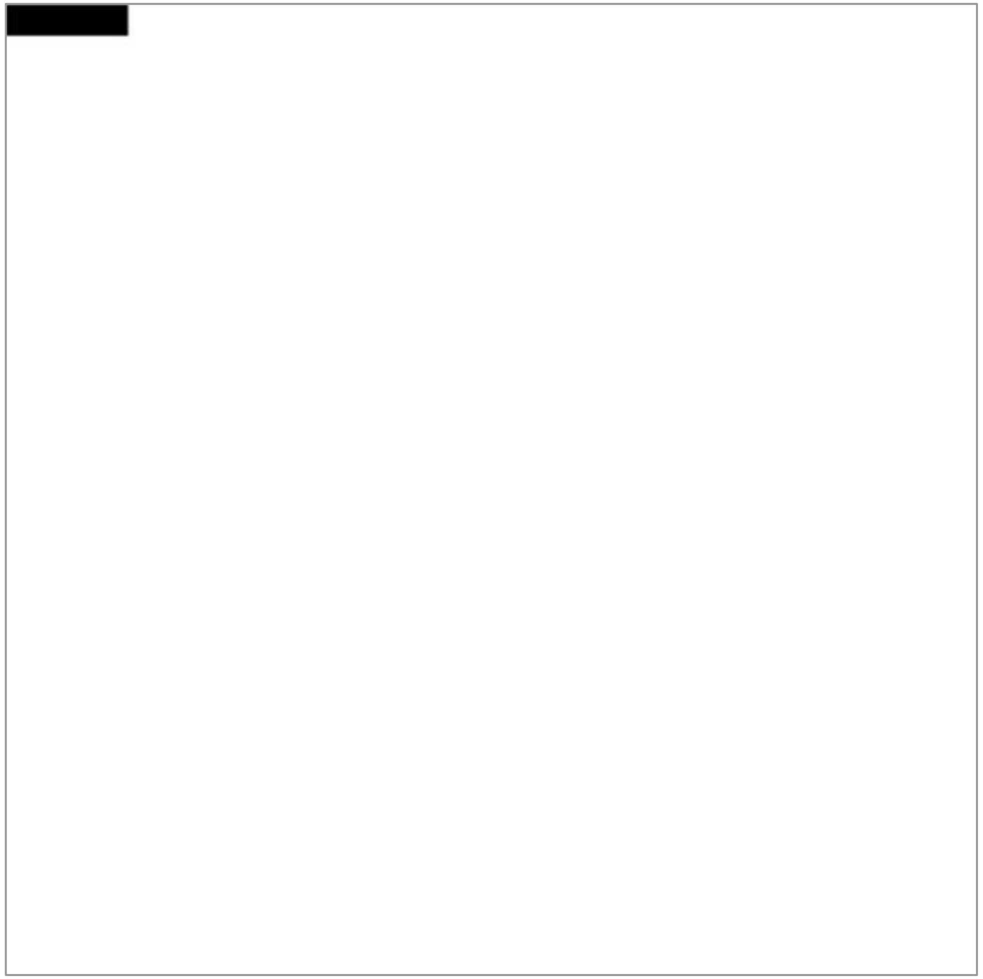
Slide by Aaron van den Oord

Softmax Sampling

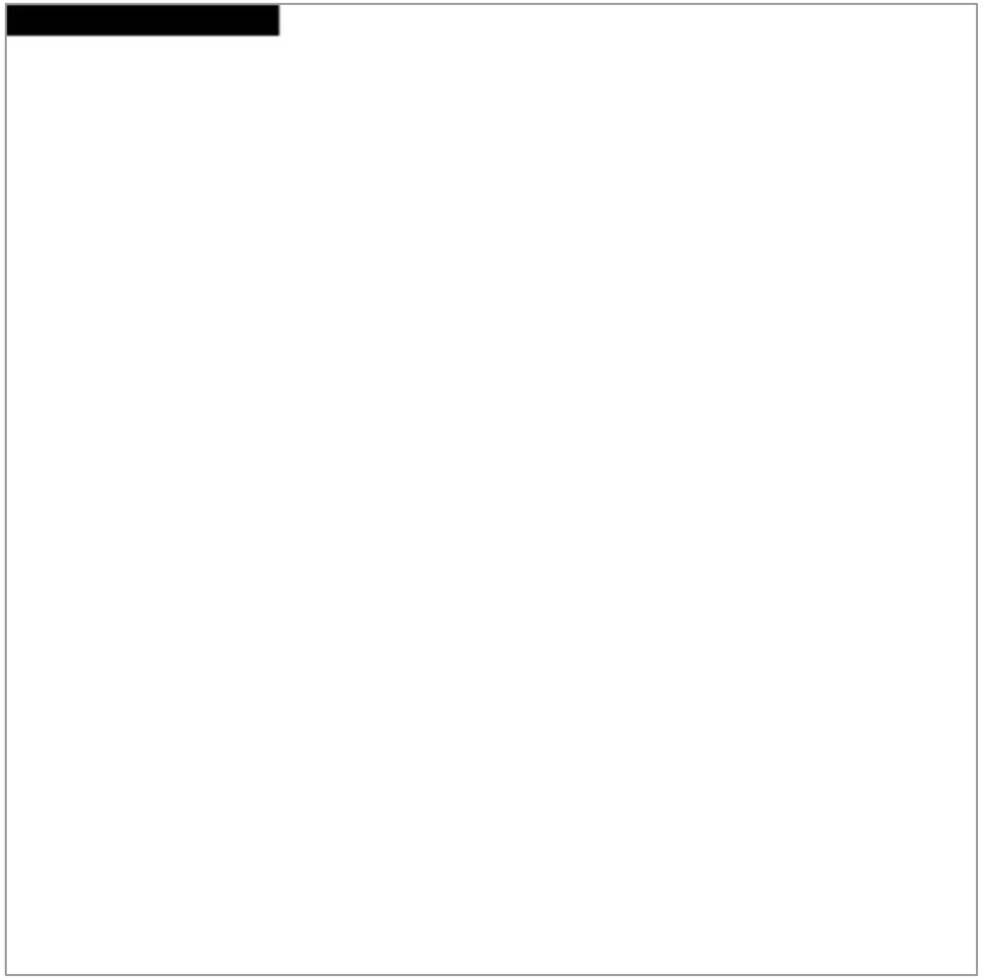


Slide by Aaron van den Oord

Softmax Sampling

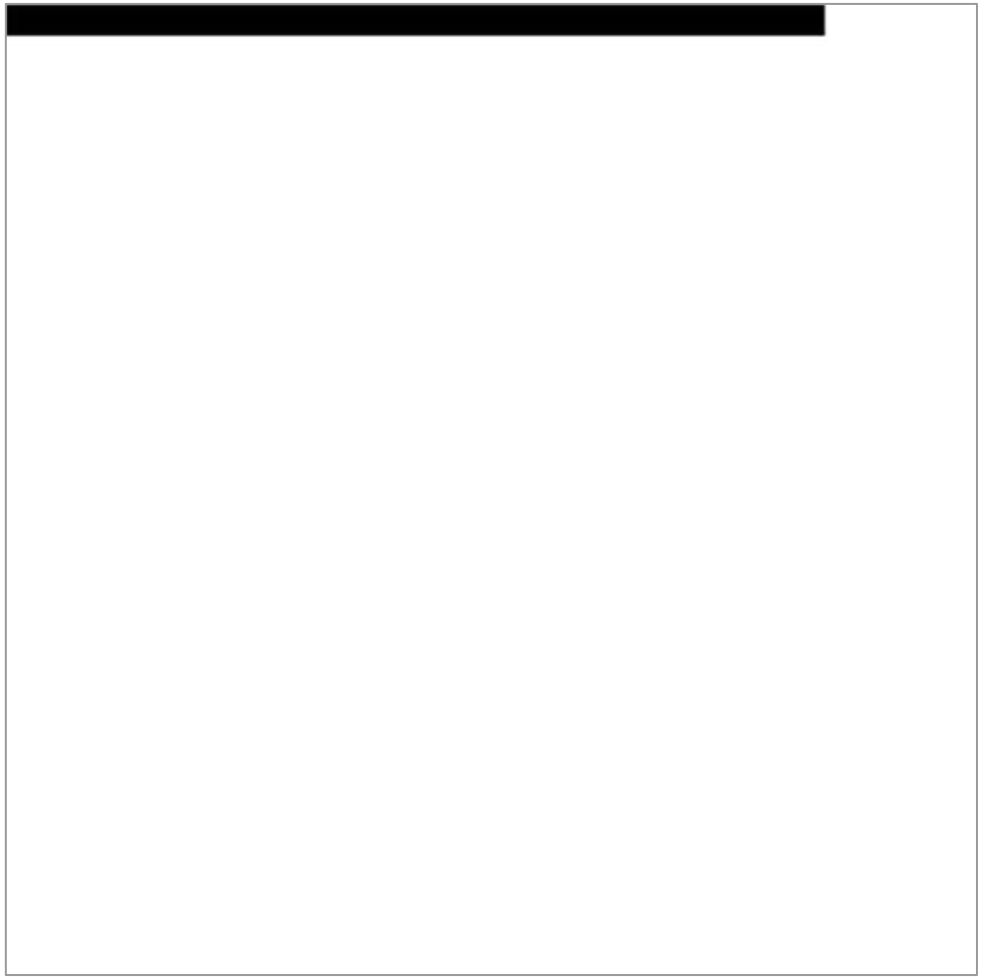


Softmax Sampling



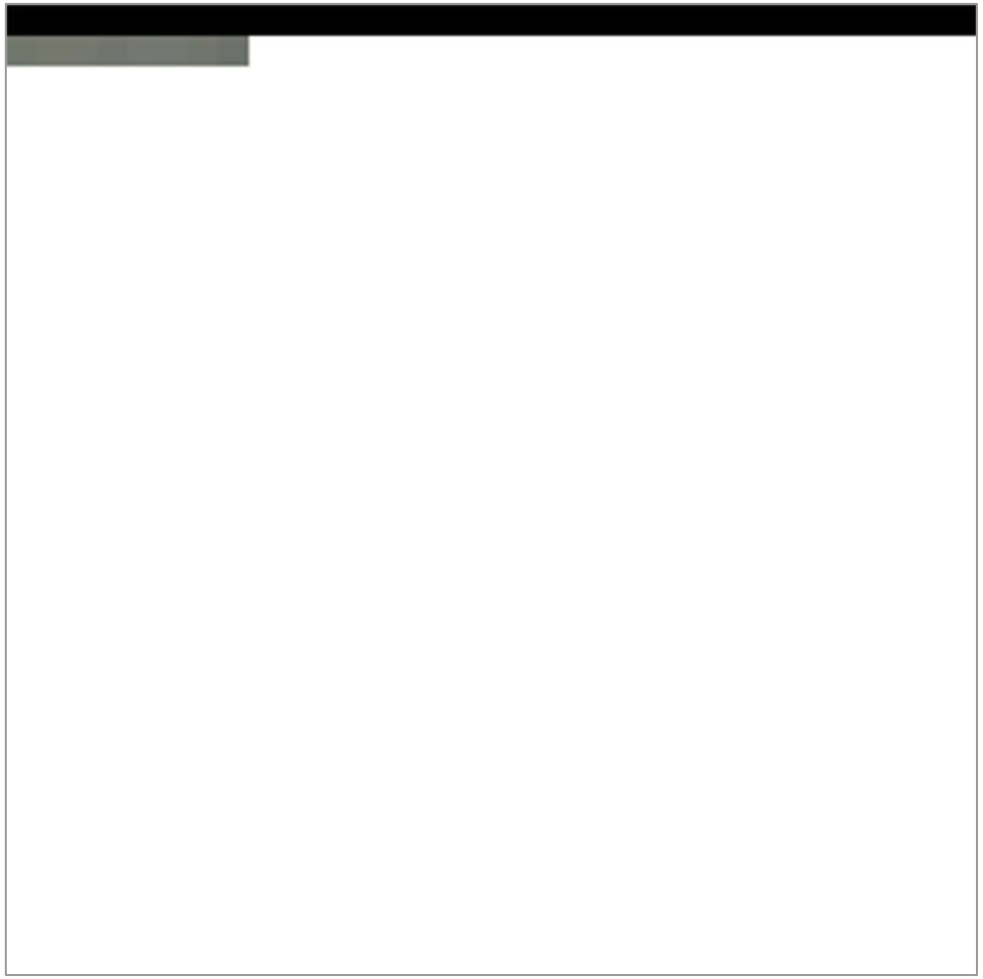
Slide by Aaron van den Oord

Softmax Sampling

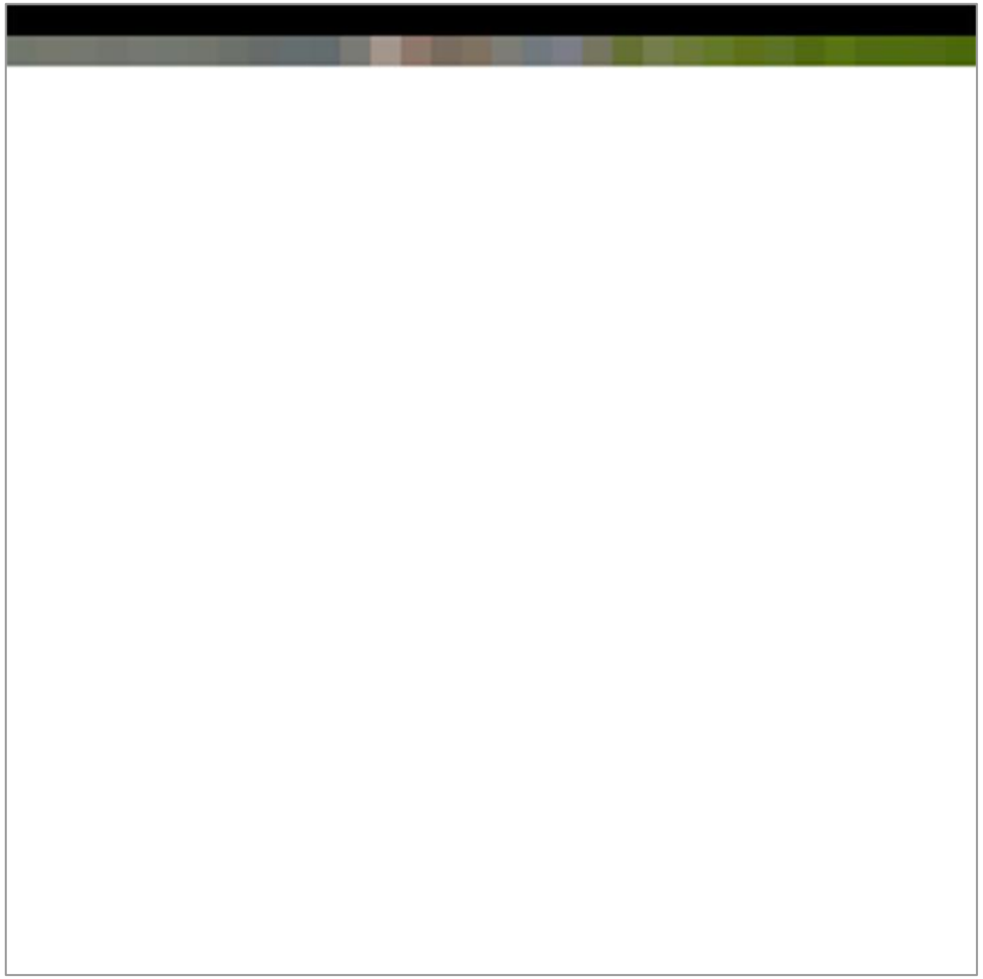


Slide by Aaron van den Oord

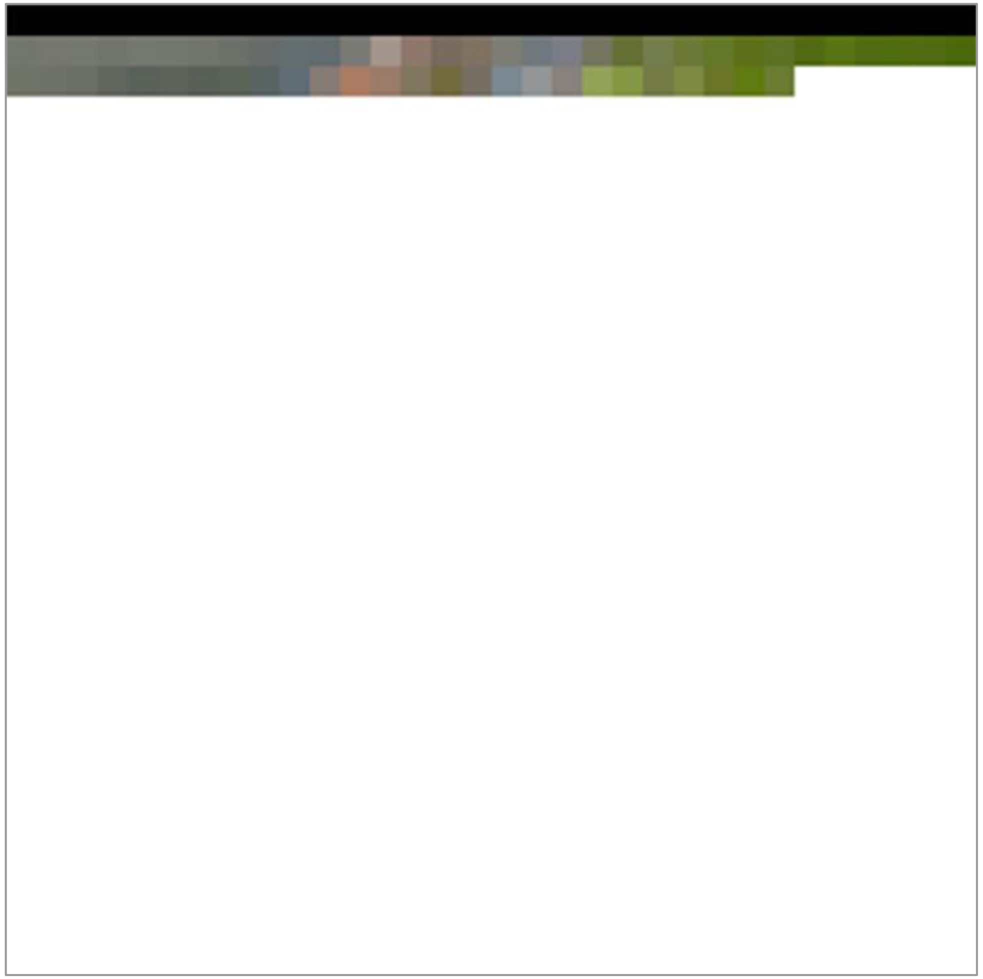
Softmax Sampling



Softmax Sampling

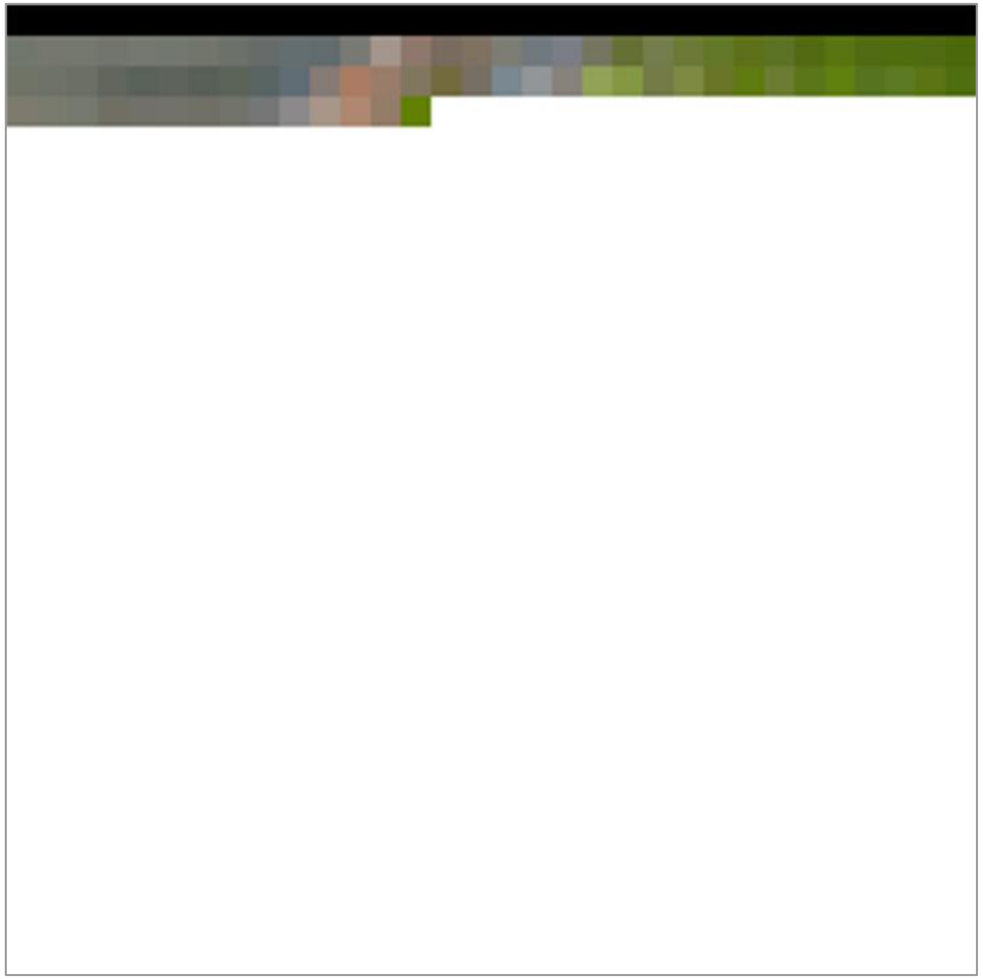


Softmax Sampling



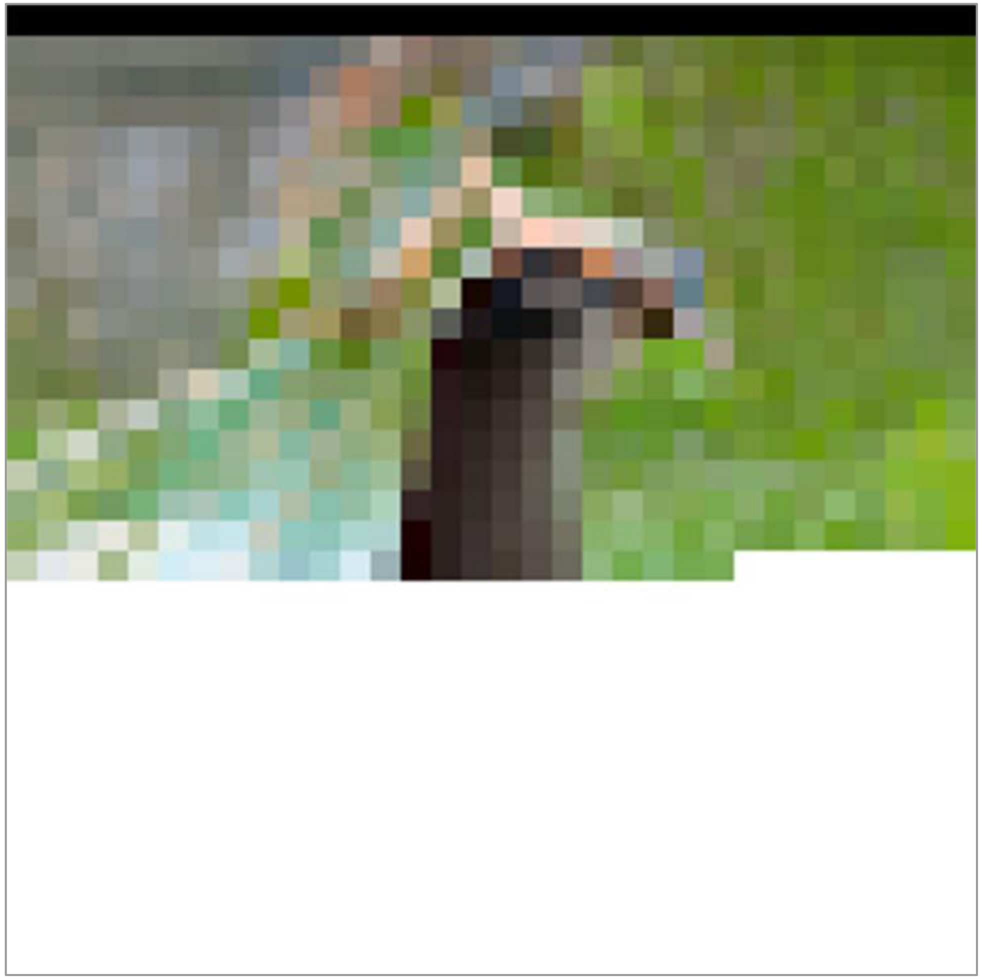
Slide by Aaron van den Oord

Softmax Sampling

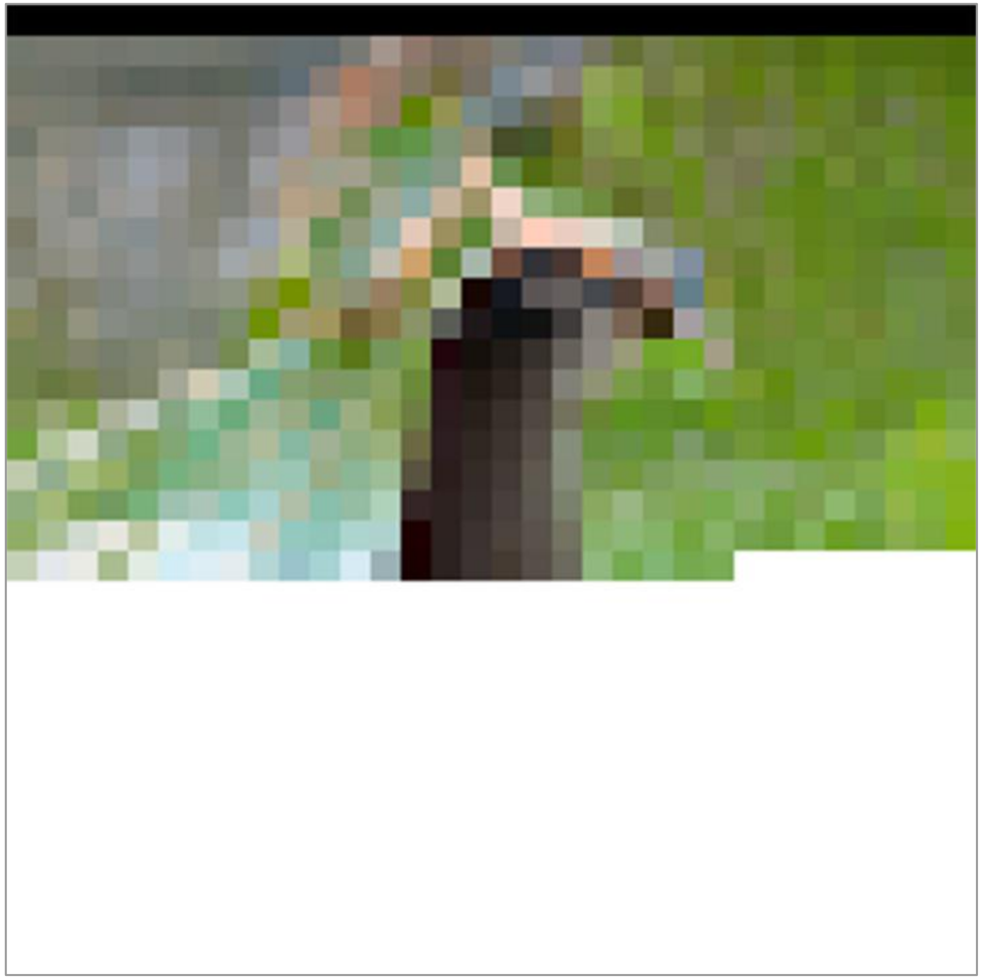


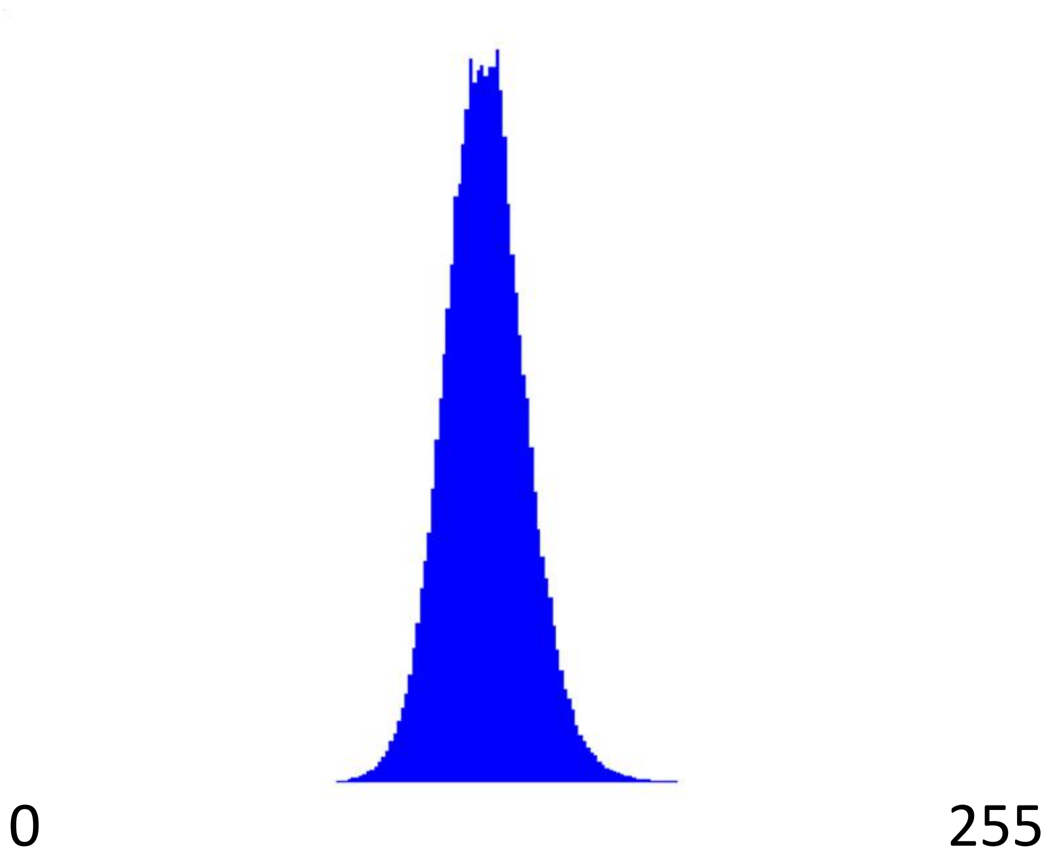
Slide by Aaron van den Oord

Softmax Sampling

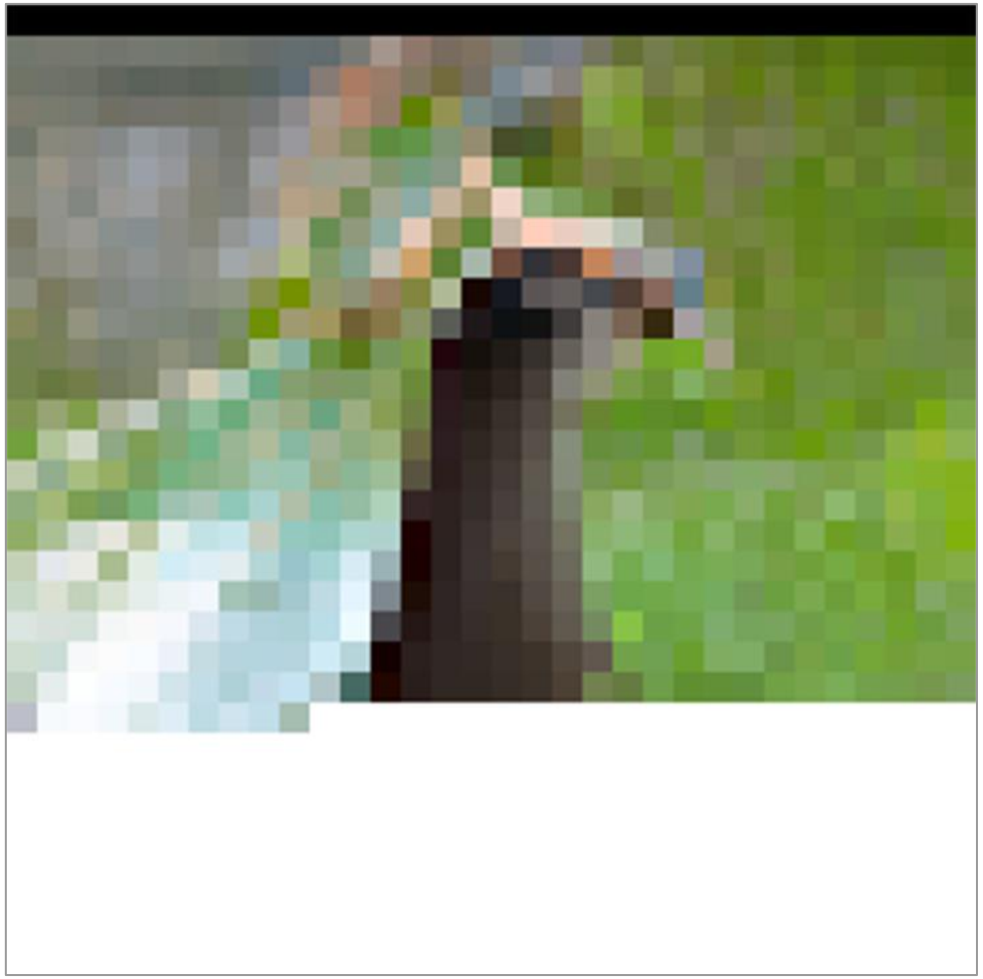


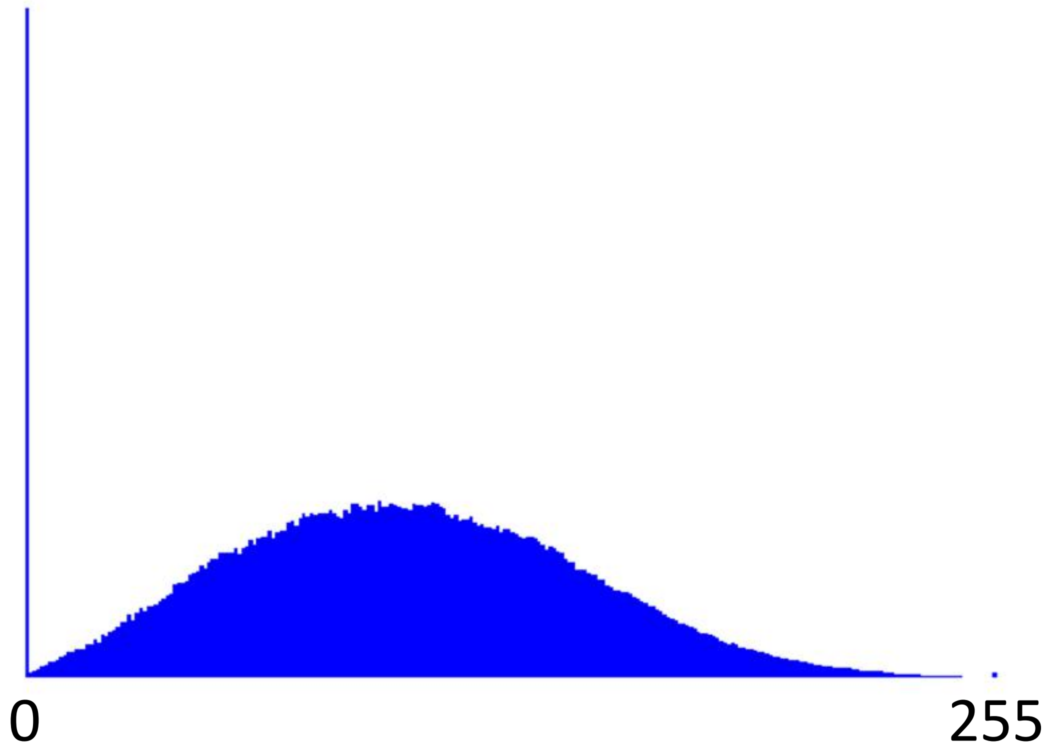
Softmax Sampling



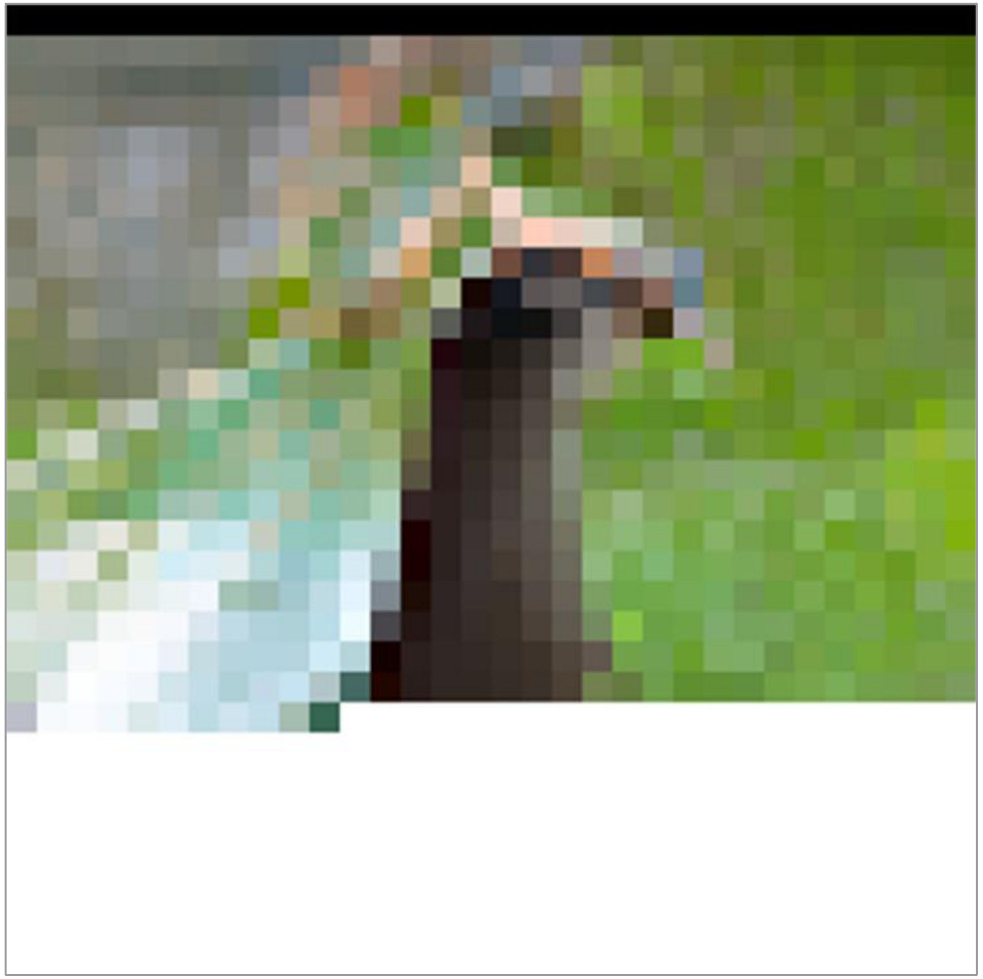


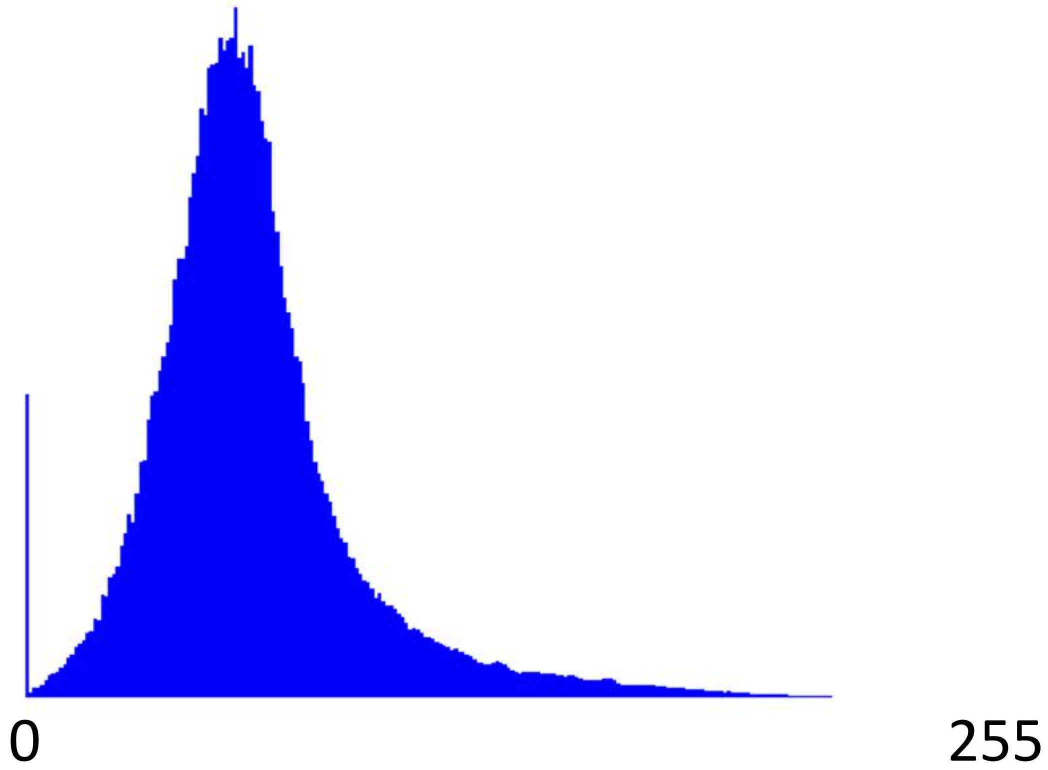
Softmax Sampling



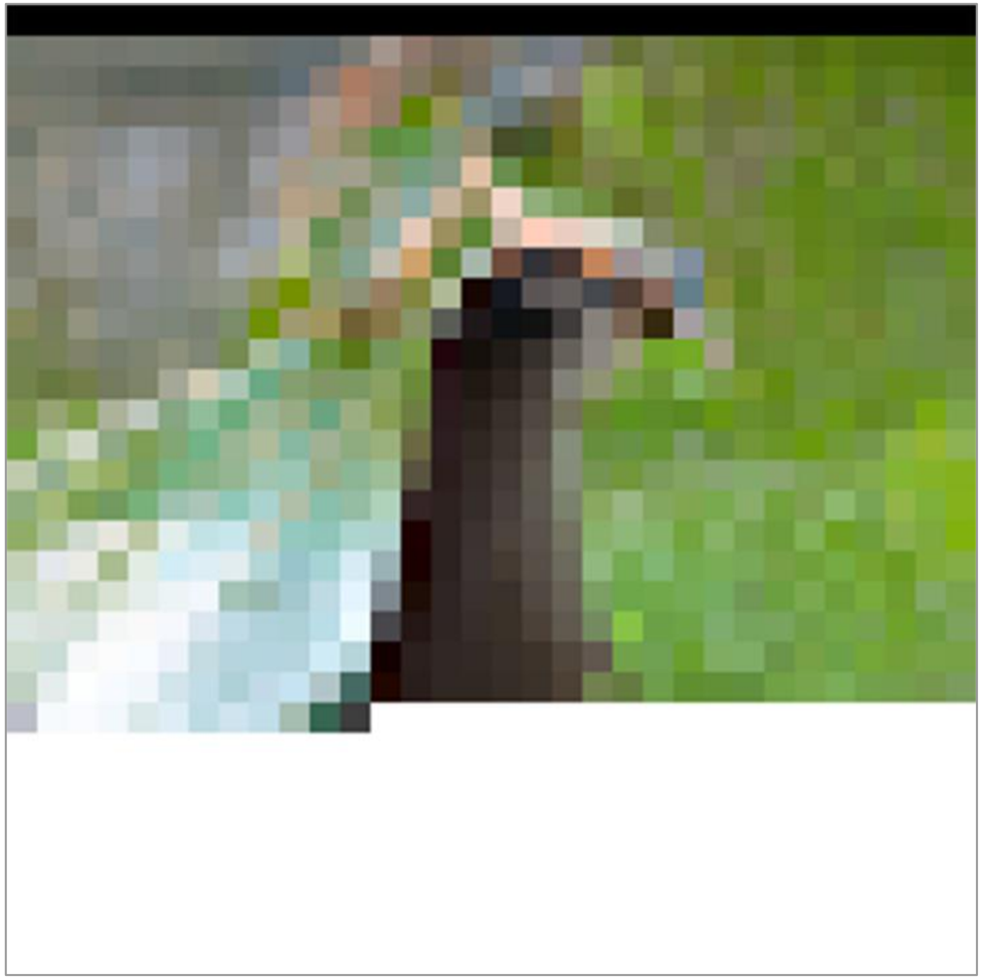


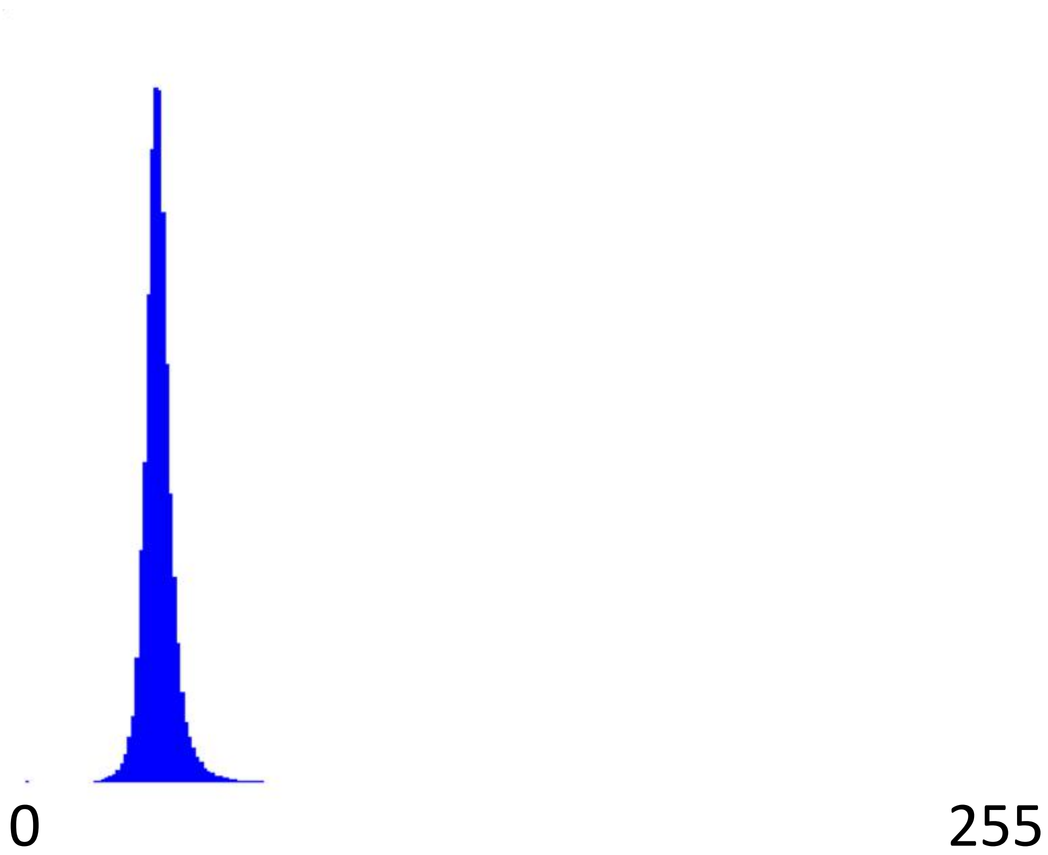
Softmax Sampling



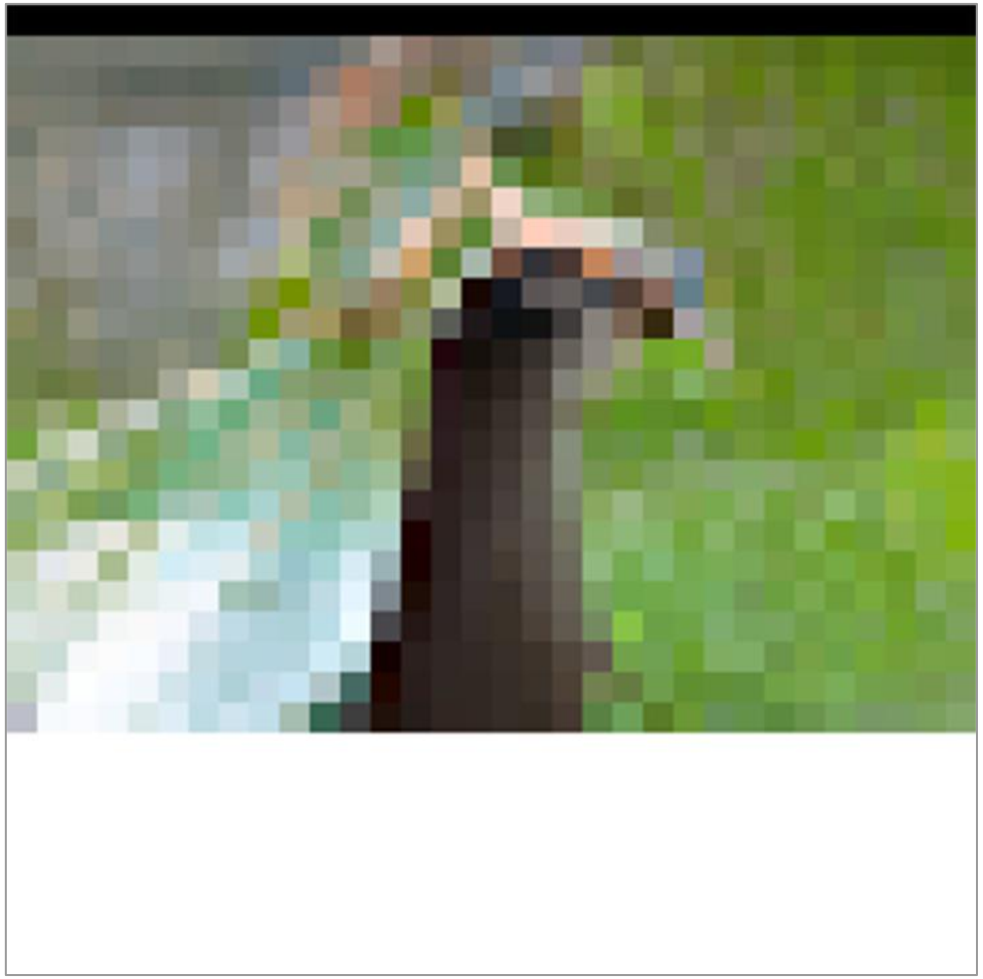


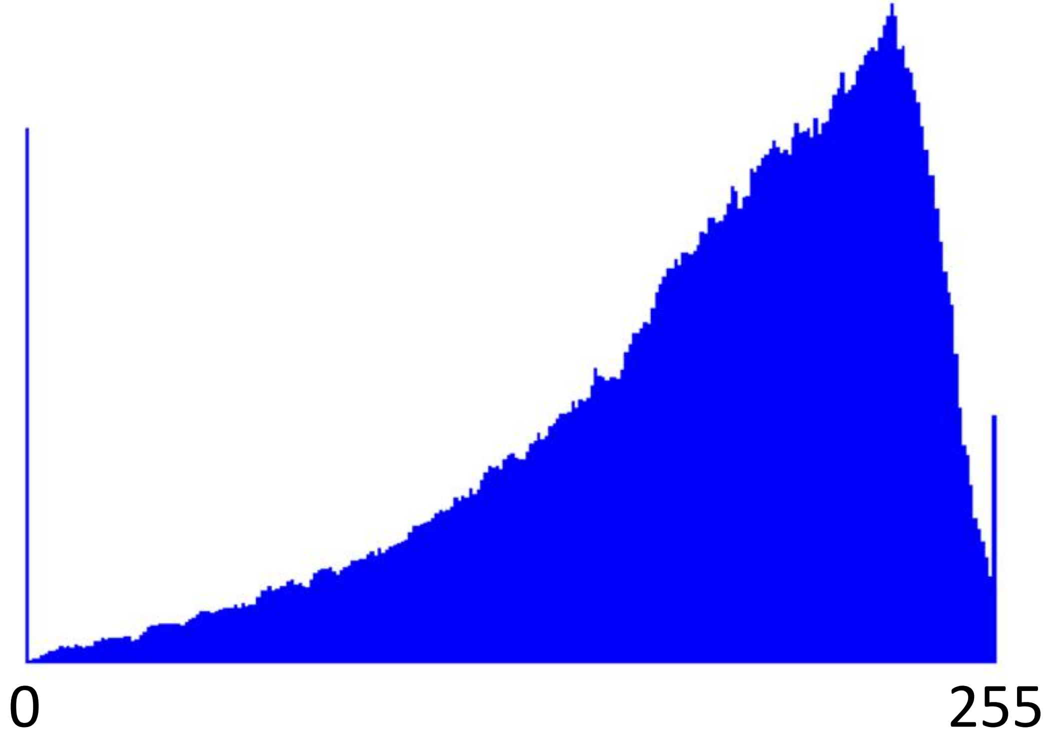
Softmax Sampling





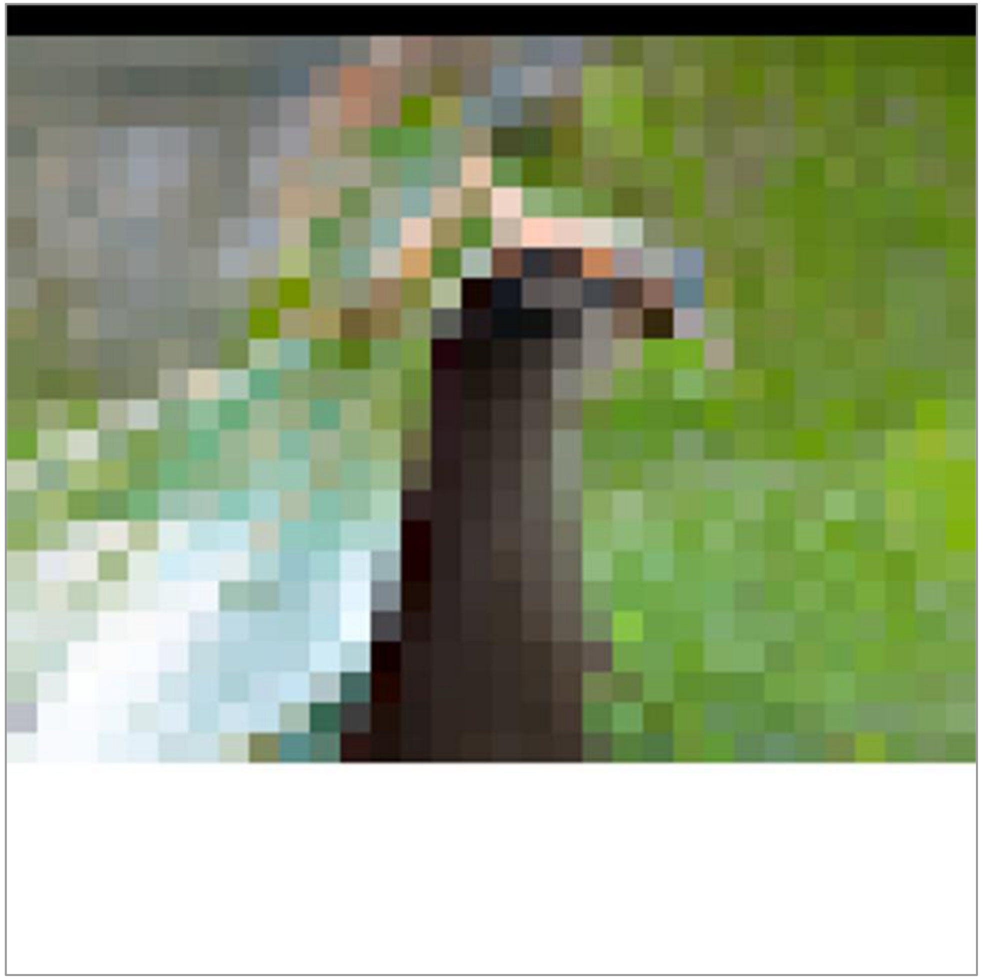
Softmax Sampling

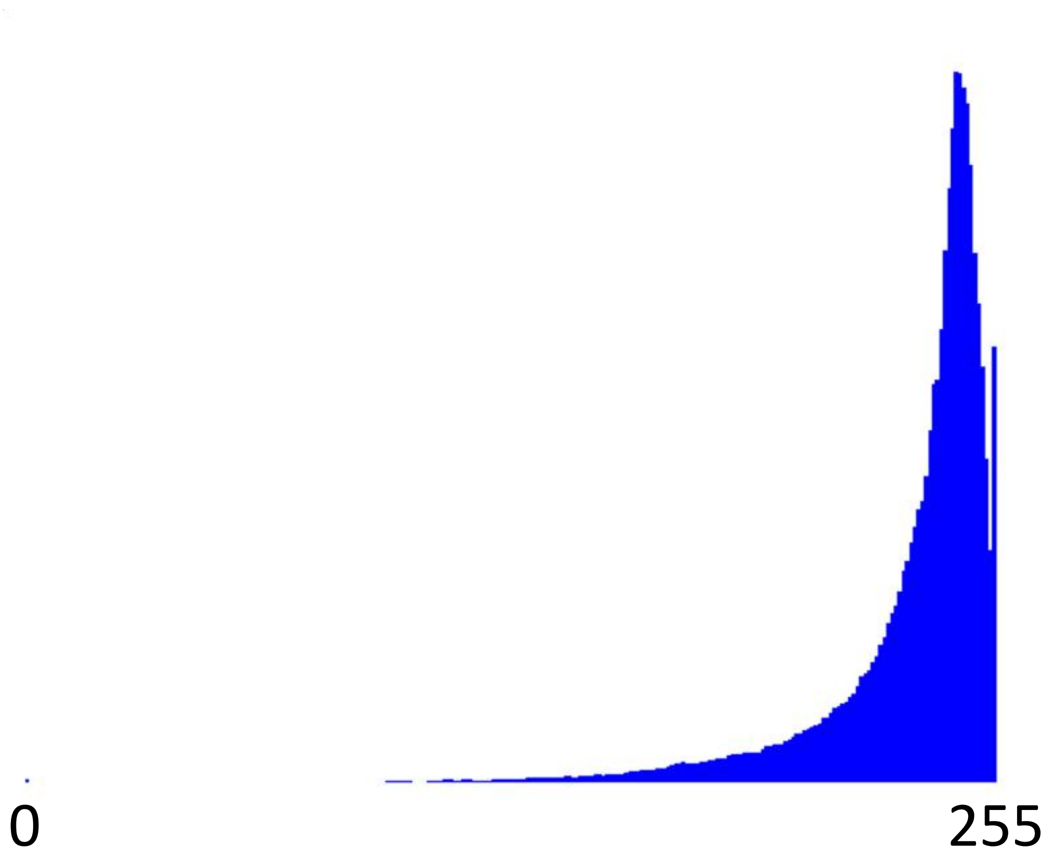




Slide by Aaron van den Oord

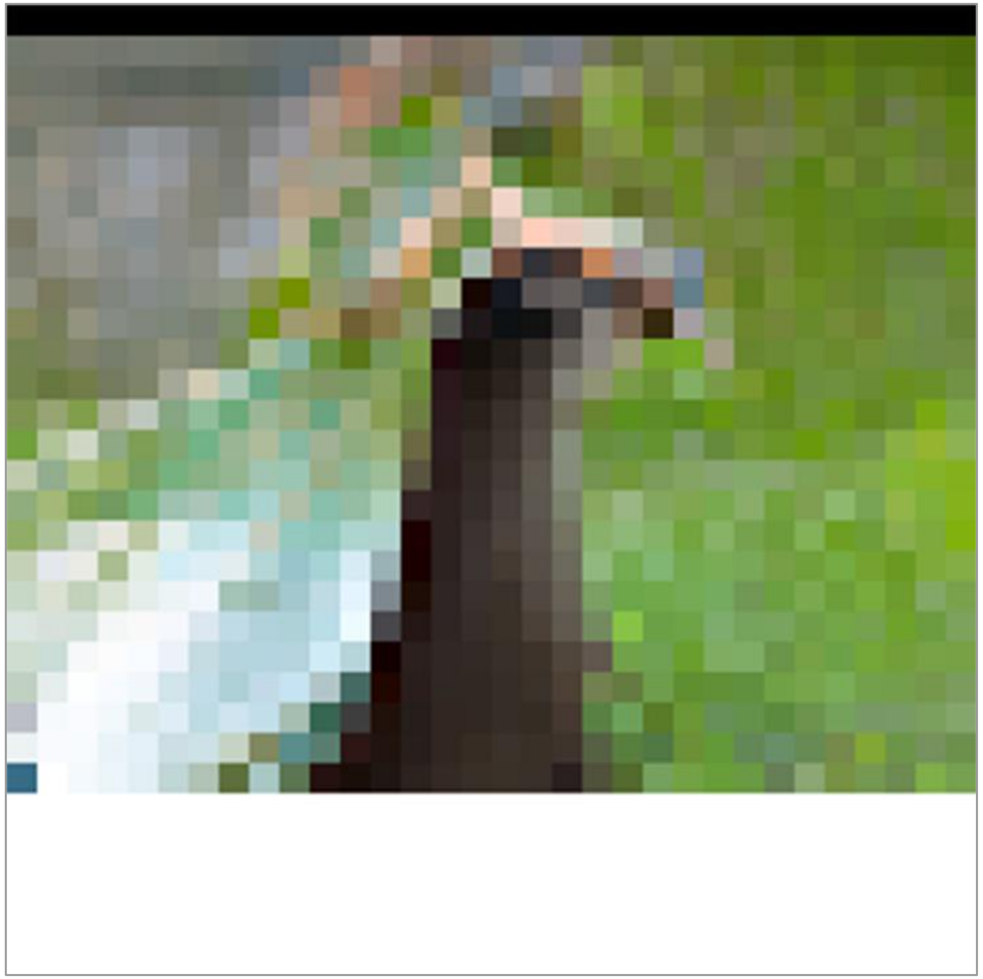
Softmax Sampling

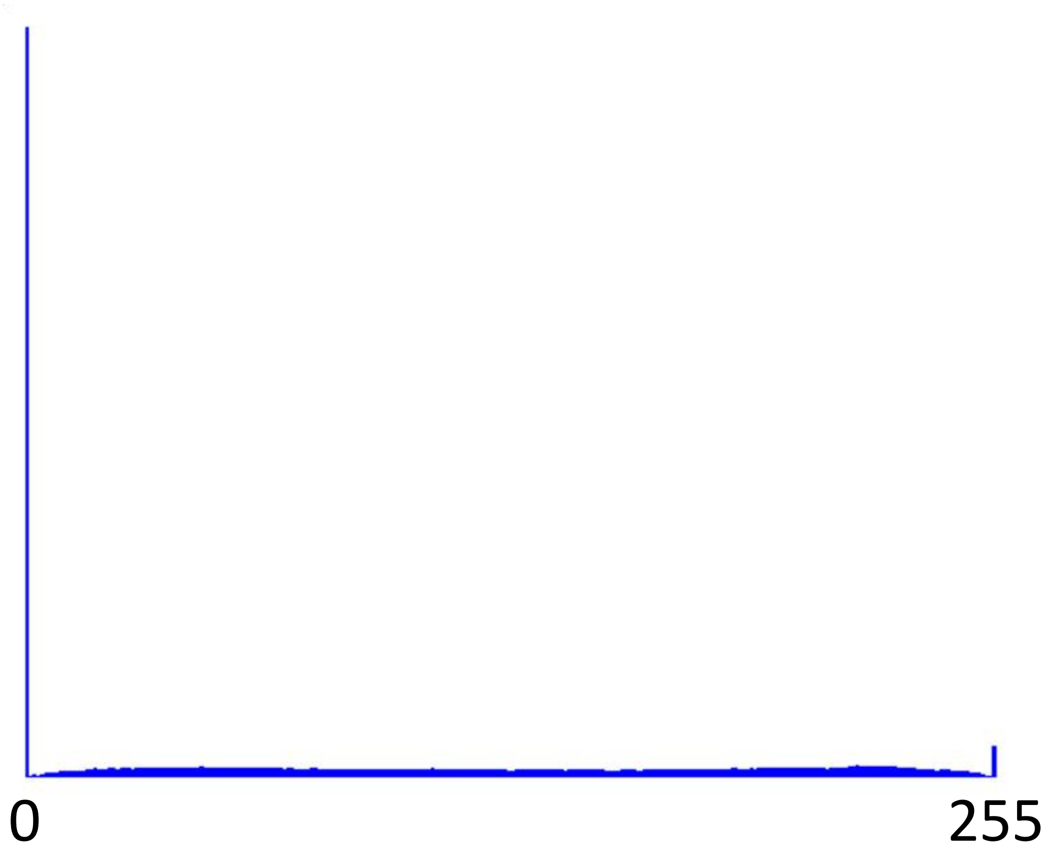




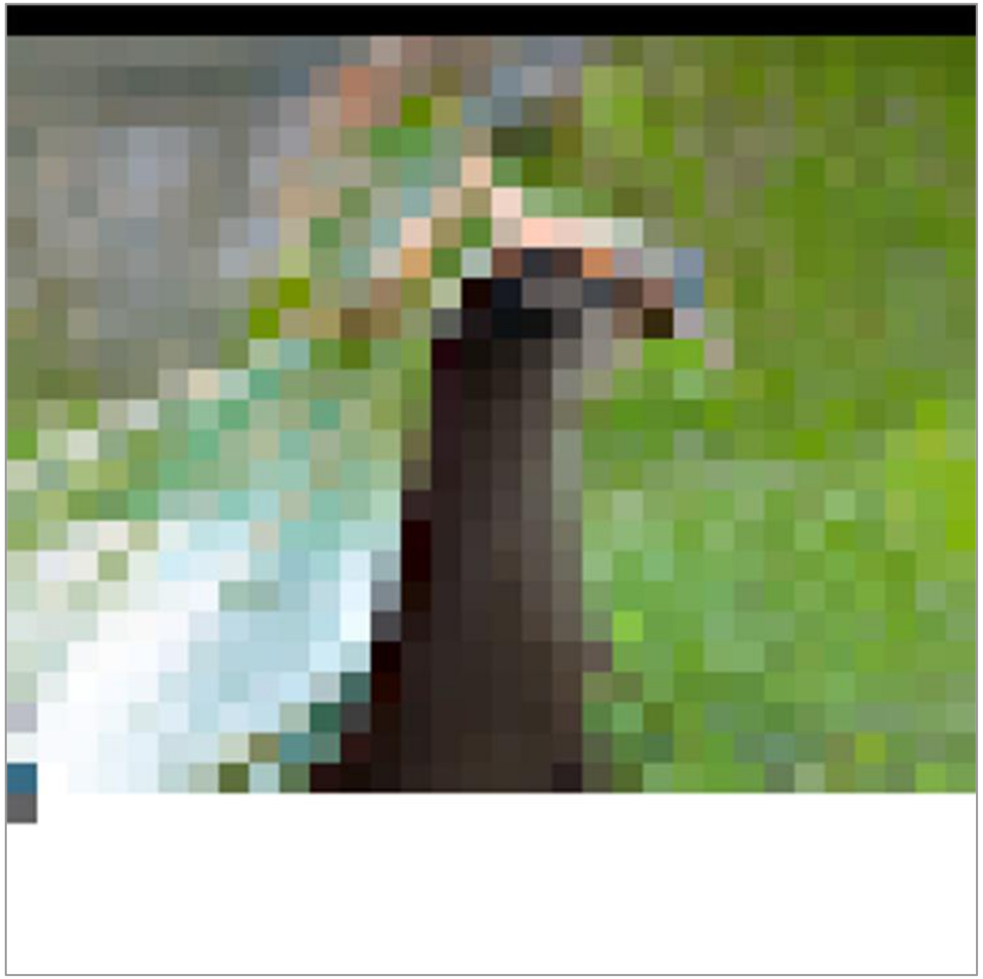
Slide by Aaron van den Oord

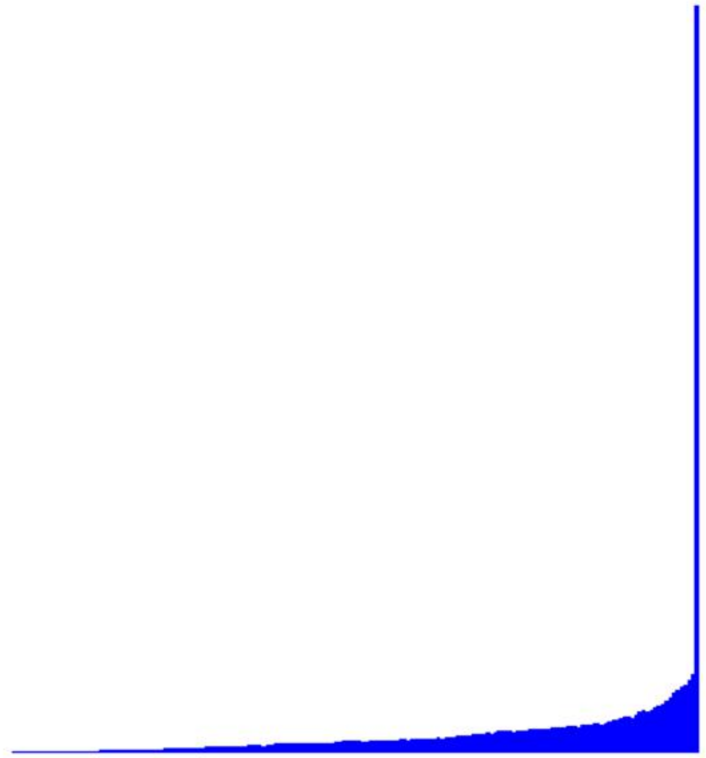
Softmax Sampling





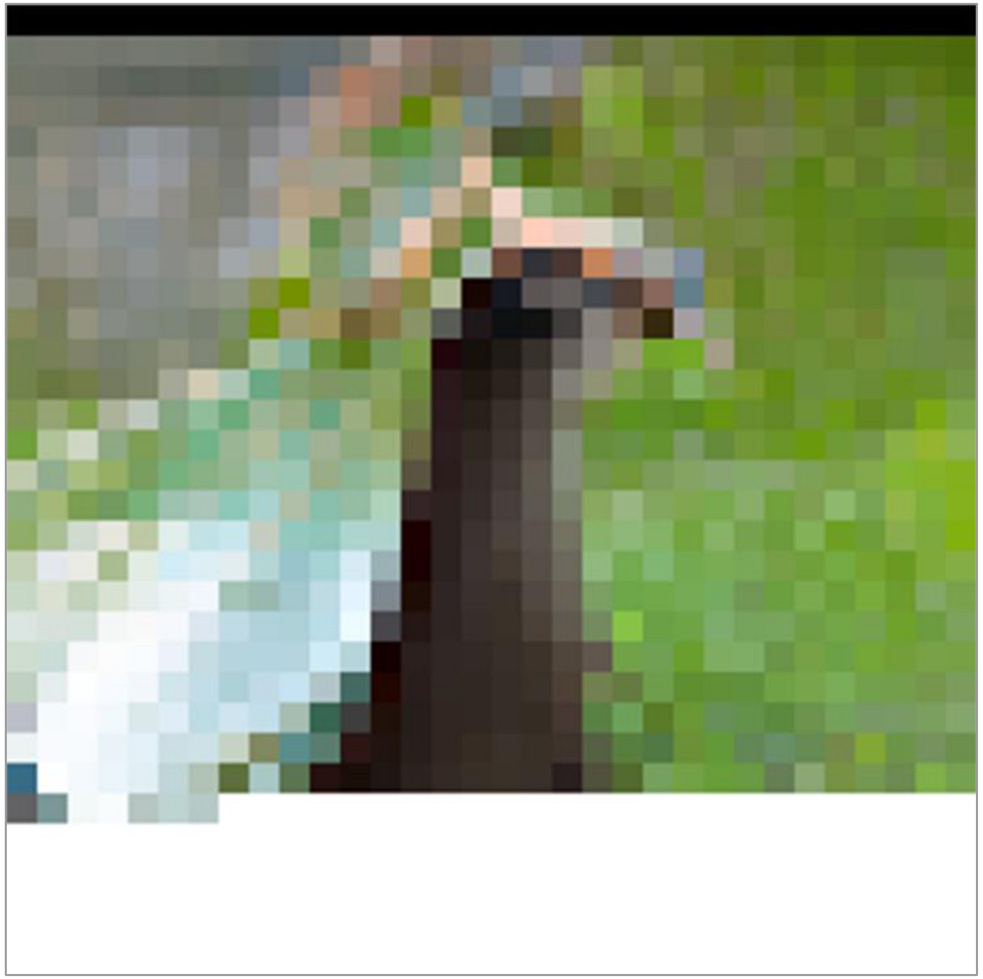
Softmax Sampling

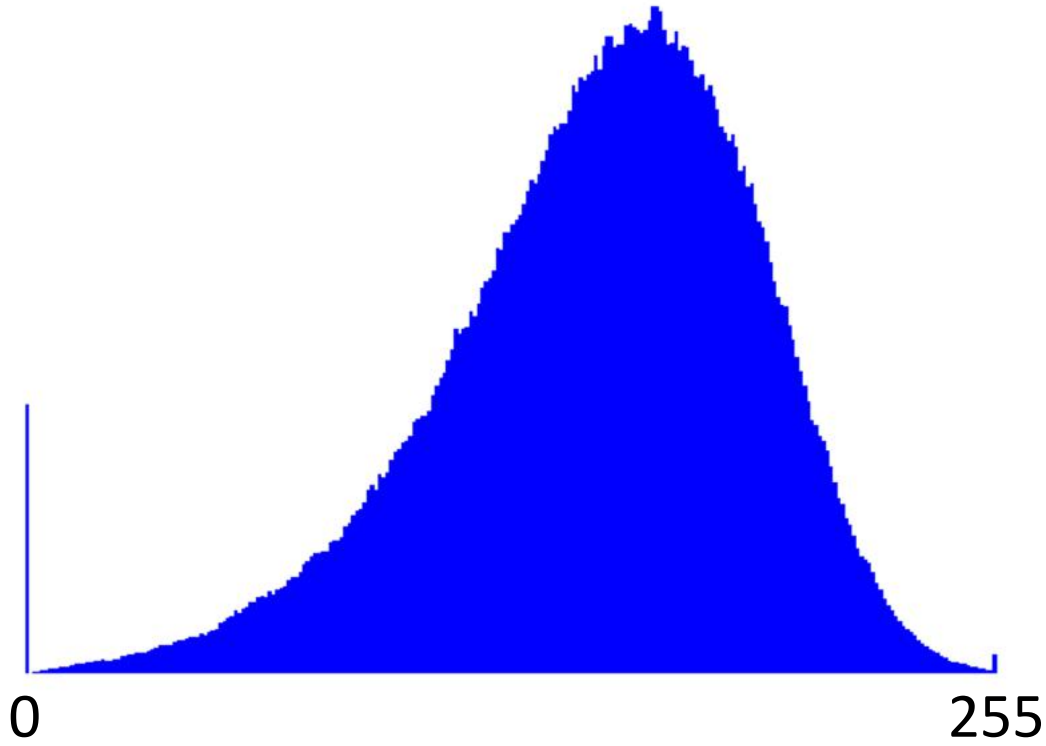




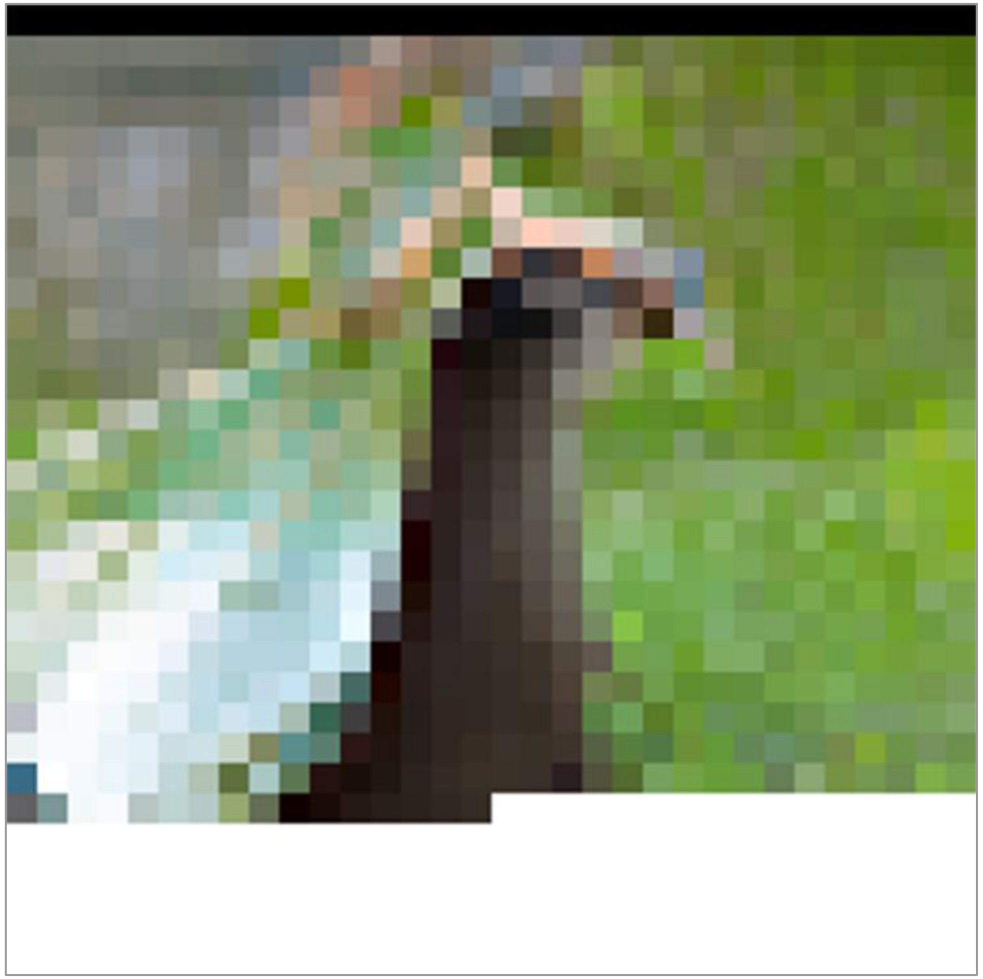
Slide by Aaron van den Oord

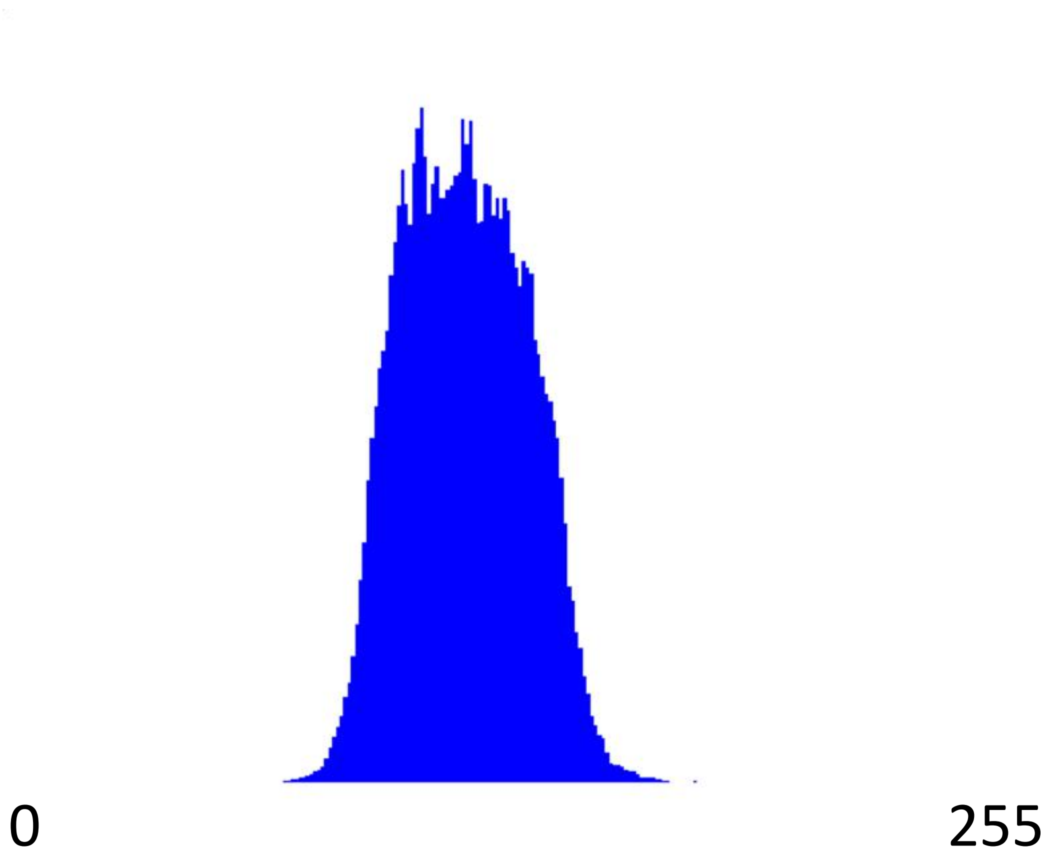
Softmax Sampling





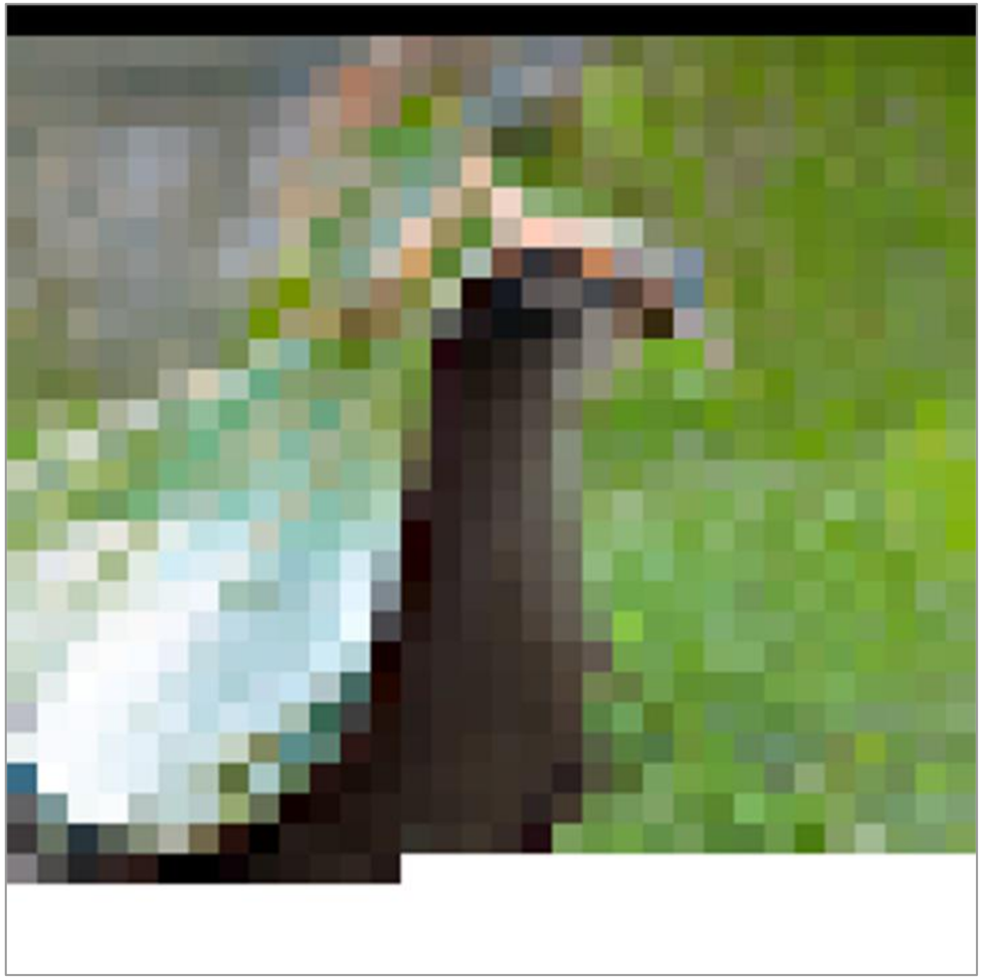
Softmax Sampling

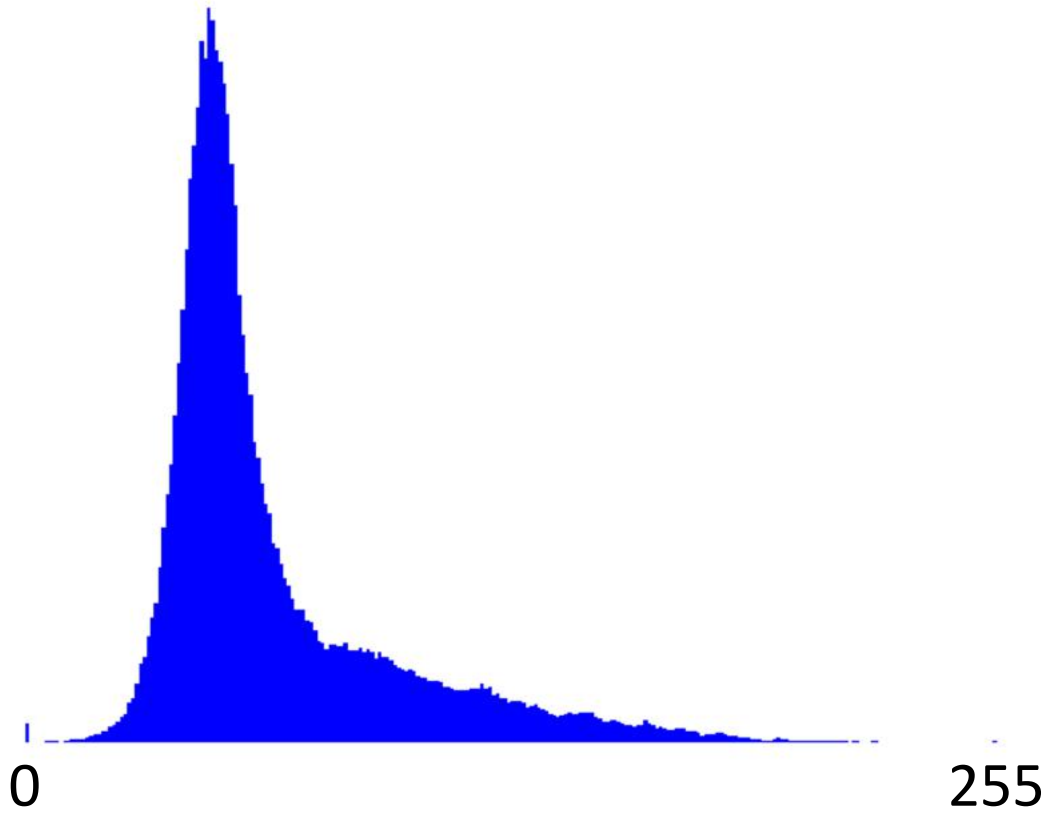




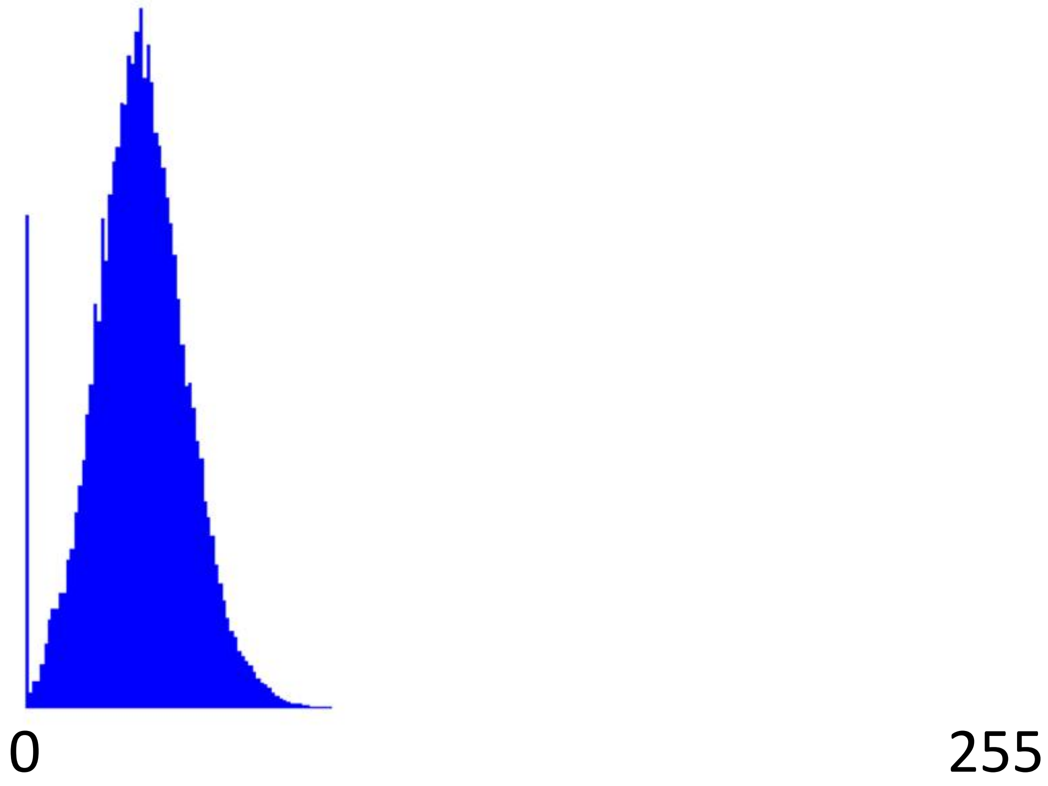
Slide by Aaron van den Oord

Softmax Sampling





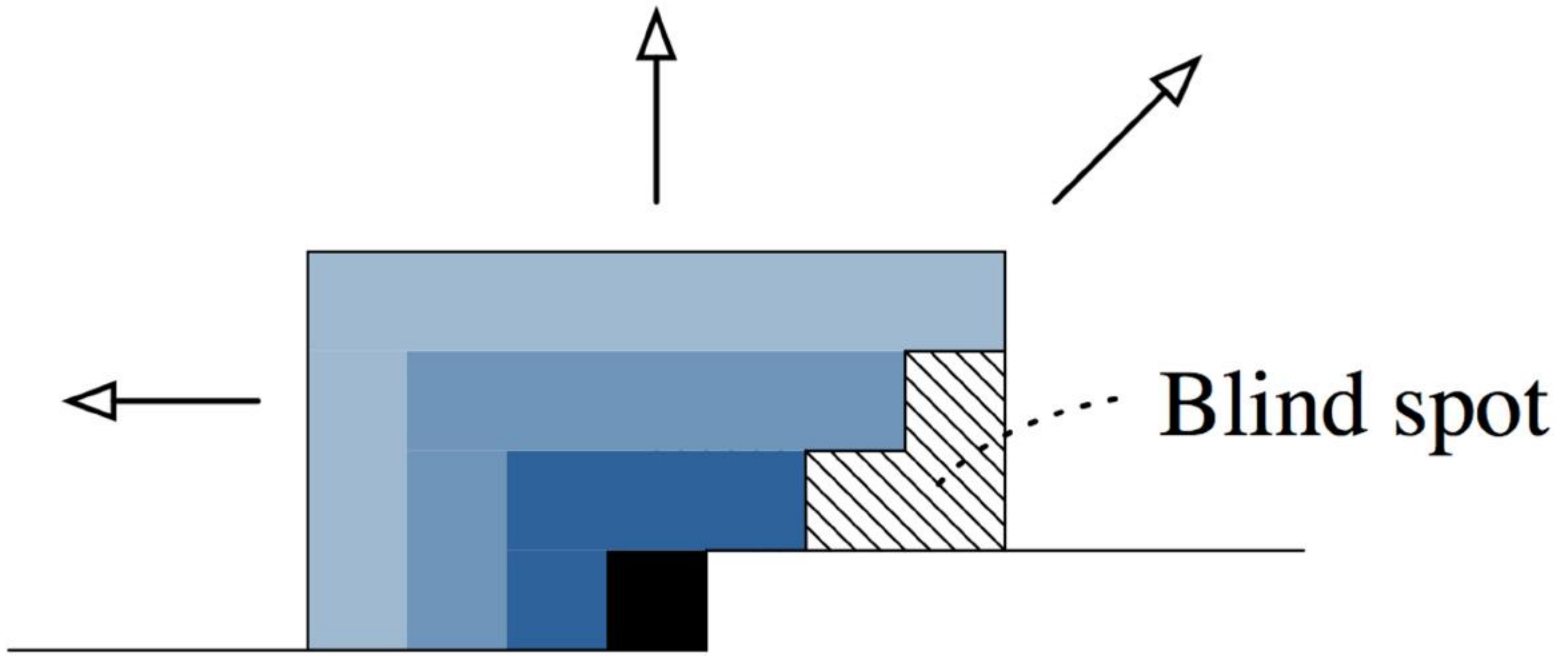
Softmax Sampling



Slide by Aaron van den Oord

PixelCNN

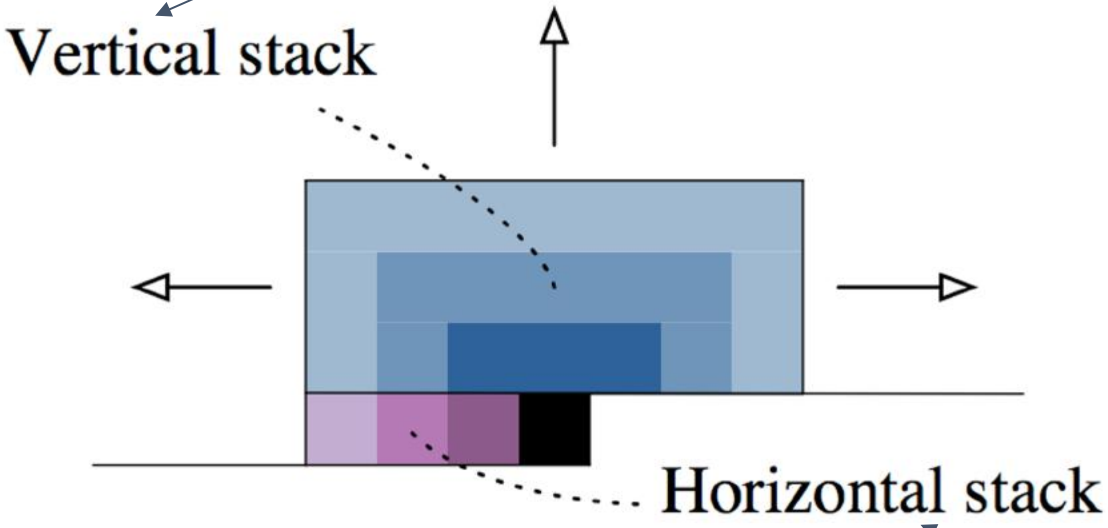
- PixelCNN-style masking has one problem: blind spot in receptive field



Gated PixelCNN

- Gated PixelCNN (2016) introduced a fix by combining two streams of convolutions

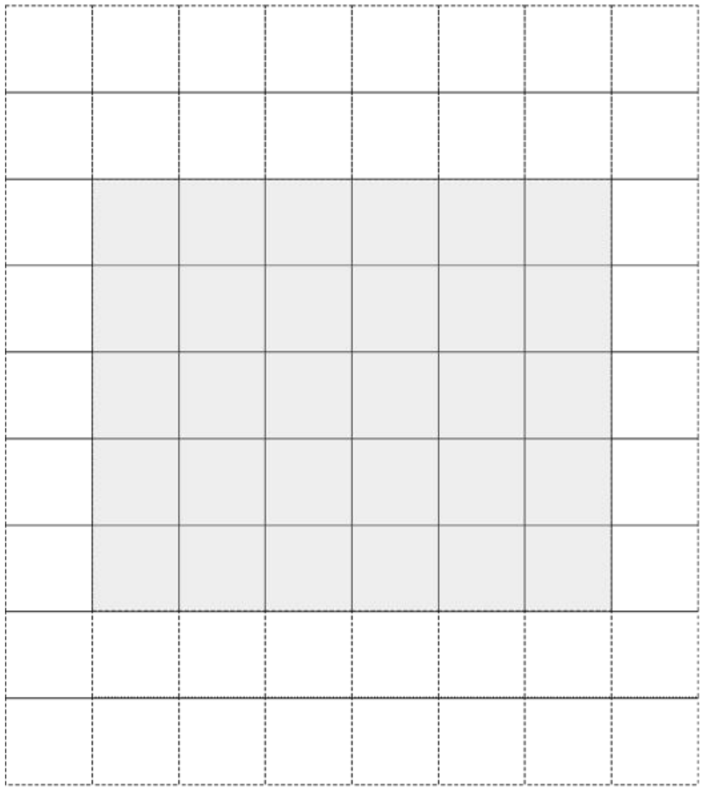
How?

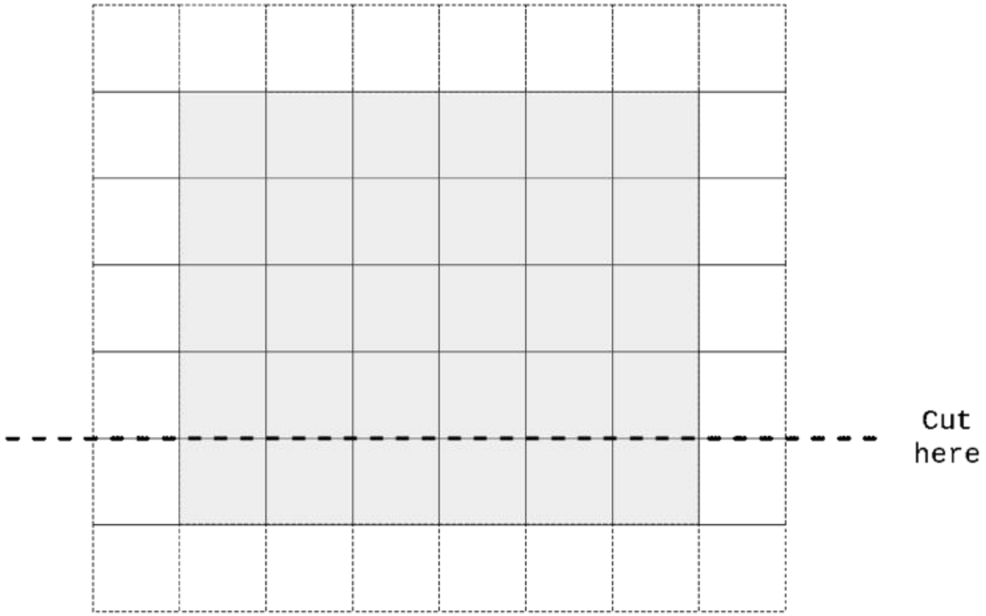


This is easy, we know how to do 1D masked conv

Gated PixelCNN

- Vertical stack: through padding, activations at ith row only depend on input before ith row

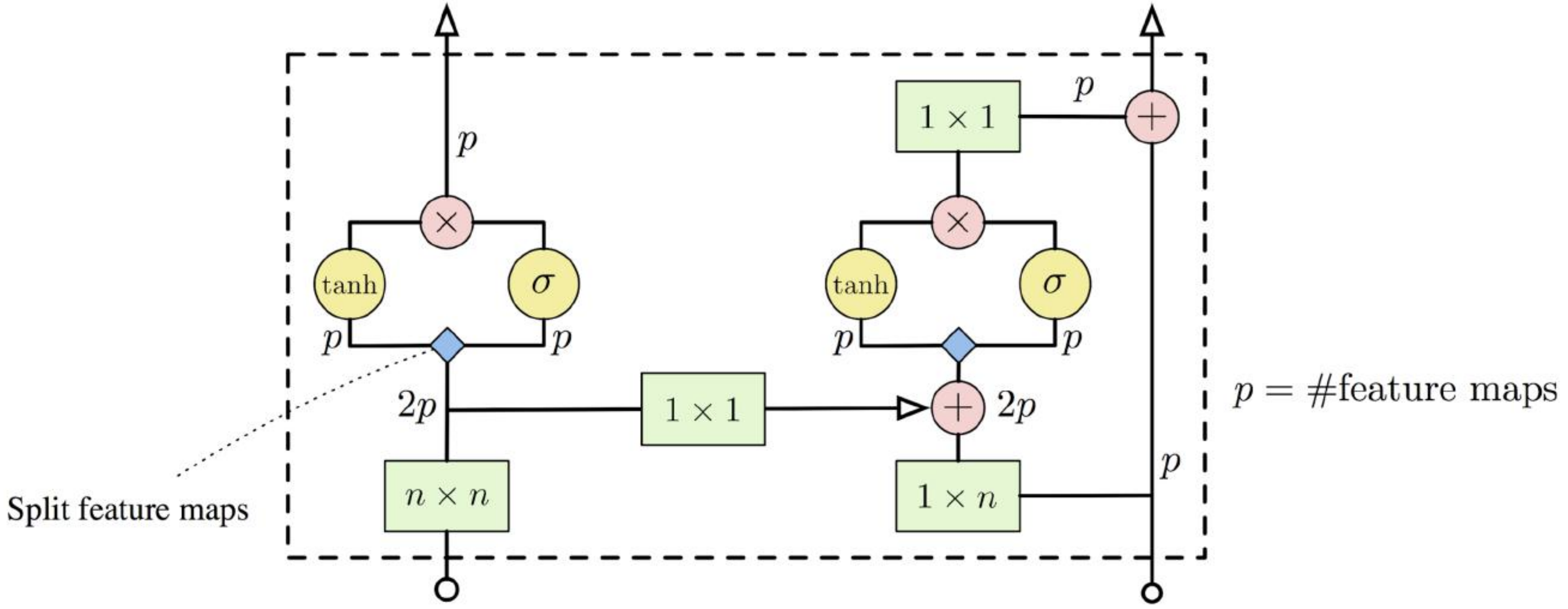




Gated PixelCNN

- Improved ConvNet architecture: Gated ResNet Block

$$\mathbf{y} = \tanh(W_{k,f} * \mathbf{x}) \odot \sigma(W_{k,g} * \mathbf{x})$$



Gated PixelCNN

- Better receptive field + more expressive architecture = better performance

Model	NLL Test (Train)
Uniform Distribution: [30]	8.00
Multivariate Gaussian: [30]	4.70
NICE: [4]	4.48
Deep Diffusion: [24]	4.20
DRAW: [9]	4.13
Deep GMMs: [31, 29]	4.00
Conv DRAW: [8]	3.58 (3.57)
RIDE: [26, 30]	3.47
PixelCNN: [30]	3.14 (3.08)
PixelRNN: [30]	3.00 (2.93)
Gated PixelCNN:	3.03 (2.90)

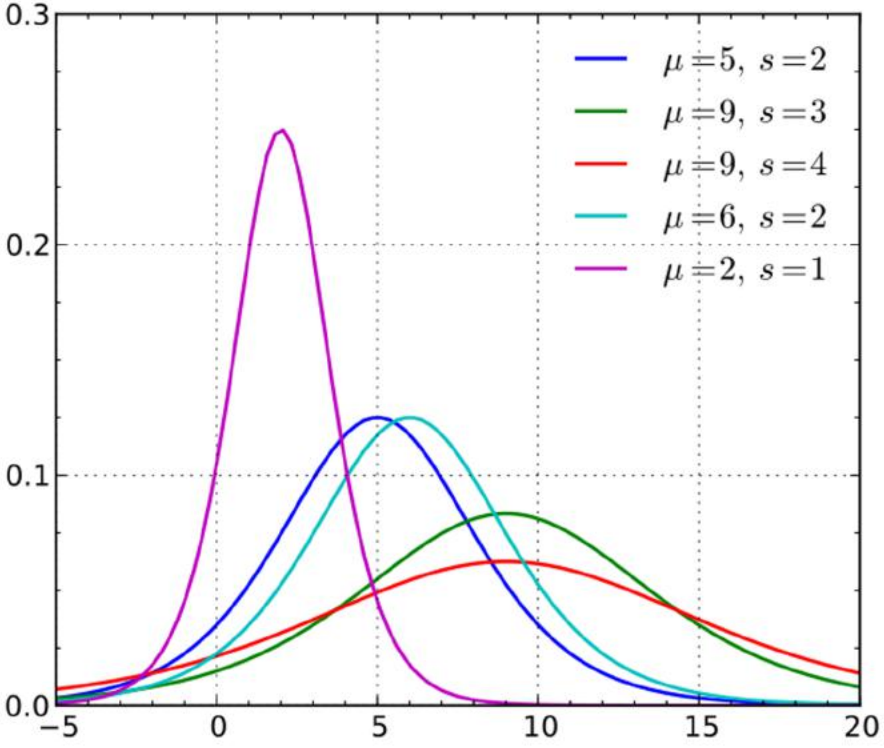
Table 1: Test set performance of different models on CIFAR-10 in *bits/dim* (lower is better), training performance in brackets.

PixelCNN++

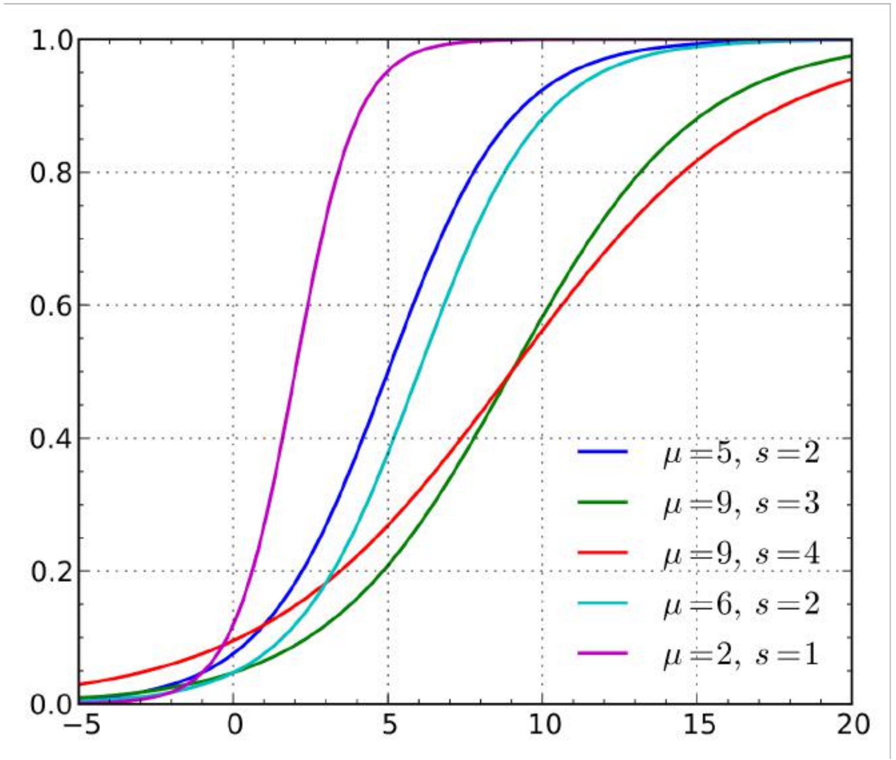
- Moving away from softmax: we know nearby pixel values are likely to co-occur!

$$\begin{aligned} \nu &\sim \sum_{i=1}^{K} \pi_i \text{logistic}(\mu_i, s_i) \\ P(x|\pi, \mu, s) &= \sum_{i=1}^{K} \pi_i \left[\sigma((x + 0.5 - \mu_i)/s_i) - \sigma((x - 0.5 - \mu_i)/s_i) \right], \end{aligned}$$

Recap: Logistic distribution



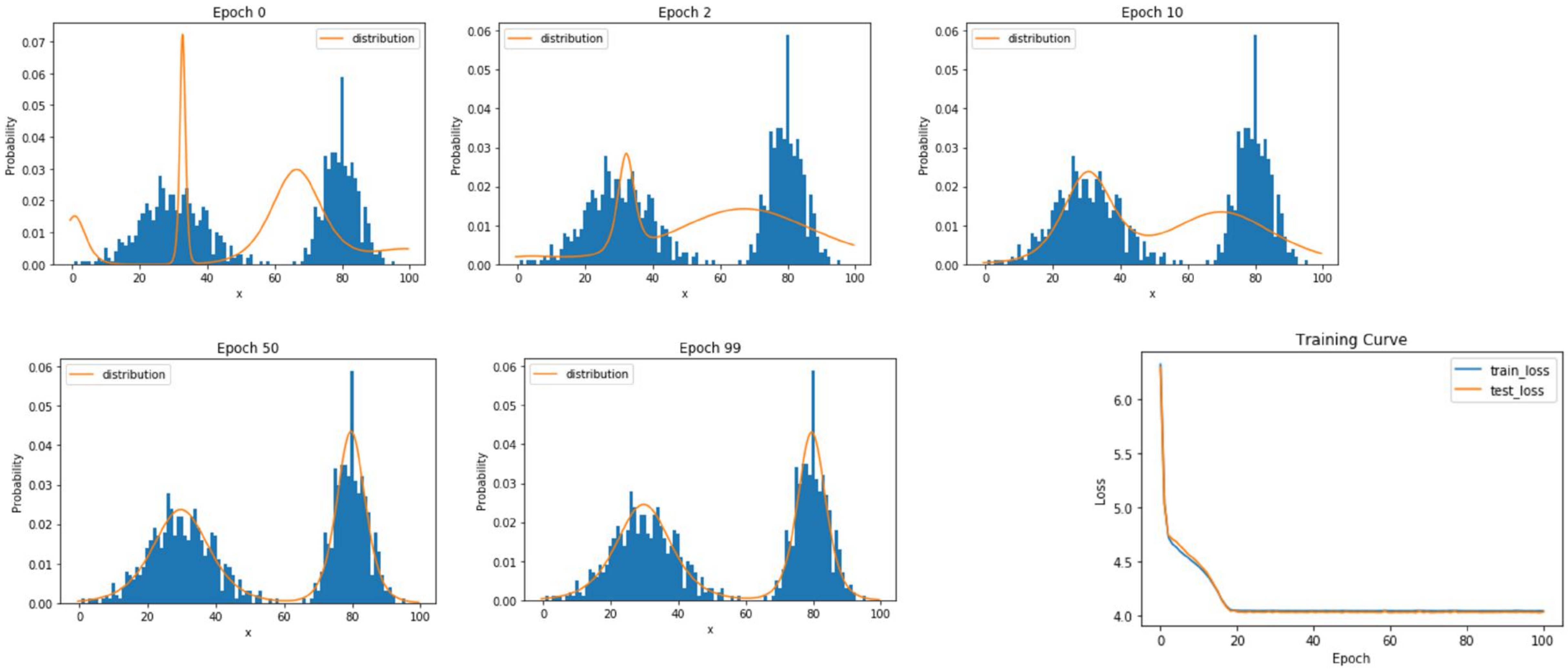
pdf



cdf

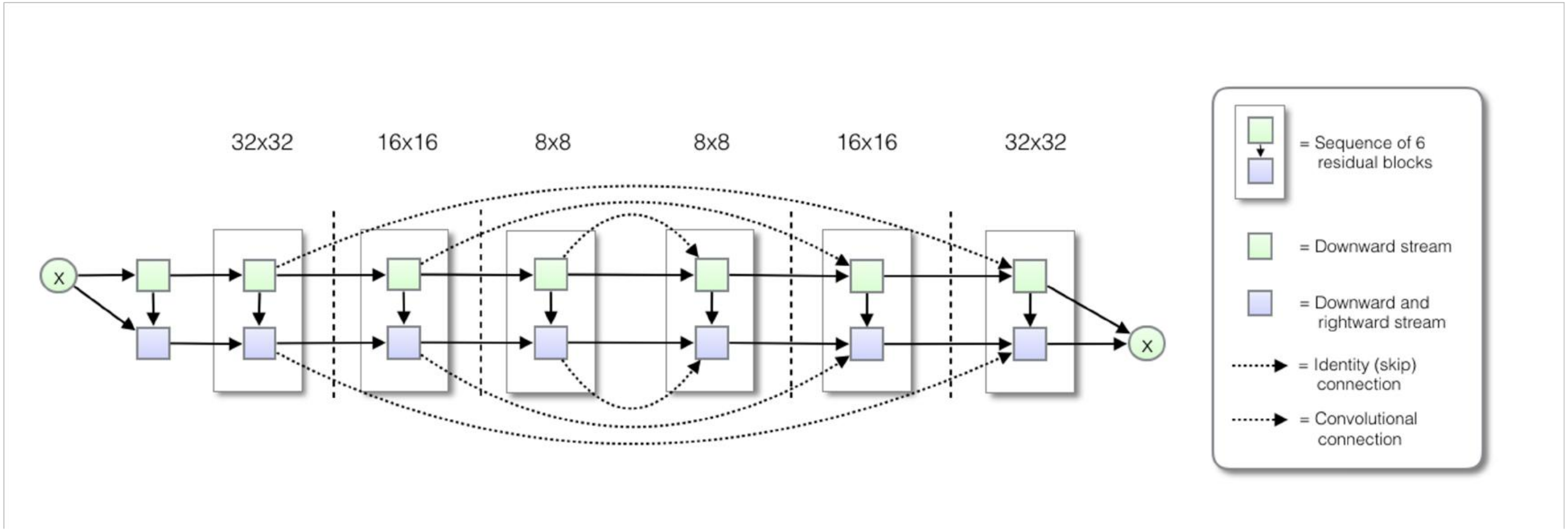
= sigmoid((x - mu) / scale)

Ex. Training Mixture of Logistics



PixelCNN++

- Capture long dependencies efficiently by downsampling



PixelCNN++

Model	Bits per sub-pixel
Deep Diffusion (Sohl-Dickstein et al., 2015)	5.40
NICE (Dinh et al., 2014)	4.48
DRAW (Gregor et al., 2015)	4.13
Deep GMMs (van den Oord & Dambre, 2015)	4.00
Conv DRAW (Gregor et al., 2016)	3.58
Real NVP (Dinh et al., 2016)	3.49
PixelCNN (van den Oord et al., 2016b)	3.14
VAE with IAF (Kingma et al., 2016)	3.11
Gated PixelCNN (van den Oord et al., 2016c)	3.03
PixelRNN (van den Oord et al., 2016b)	3.00
PixelCNN++	**2.92**

Table 1: Negative log-likelihood for generative models on CIFAR-10 expressed as bits per sub-pixel.

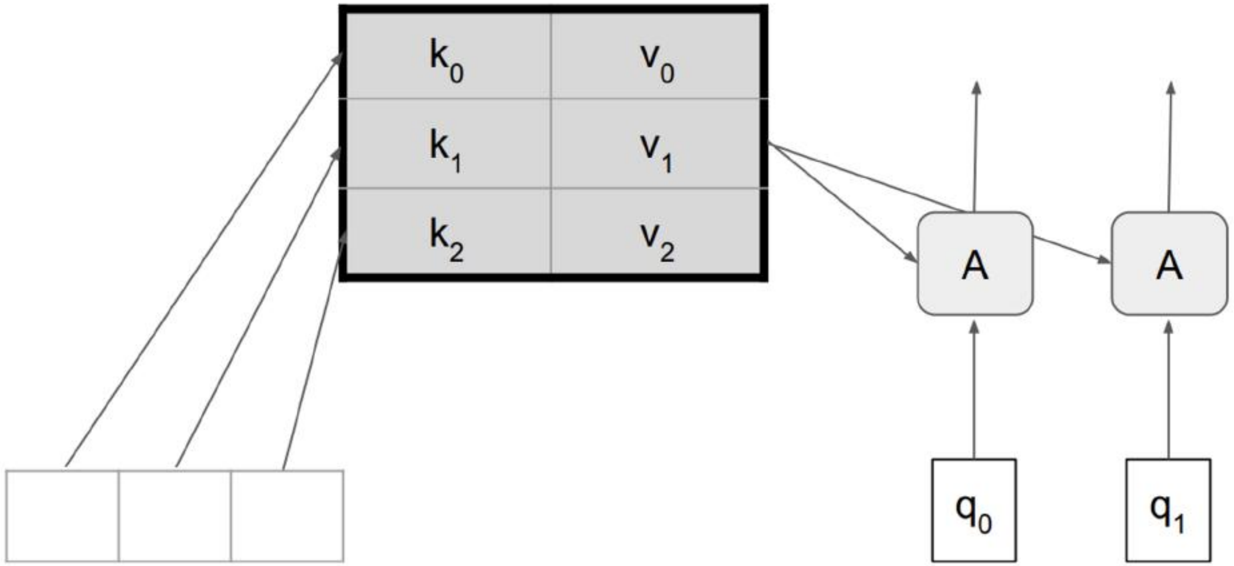
Masked Attention

- A recurring problem for convolution: limited receptive field → hard to capture long-range dependencies
- (Self-)Attention: an alternative that has
  - unlimited receptive field!!
  - also O(1) parameter scaling w.r.t. data dimension
  - parallelized computation (versus RNN)

Attention

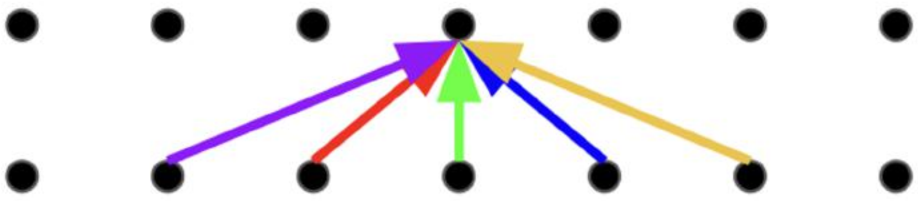
- Dot-Product Attention

$$A(q,K,V)=\sum_i \frac{e^{q\cdot k_i}}{\sum_j e^{q\cdot k_j}} v_i$$

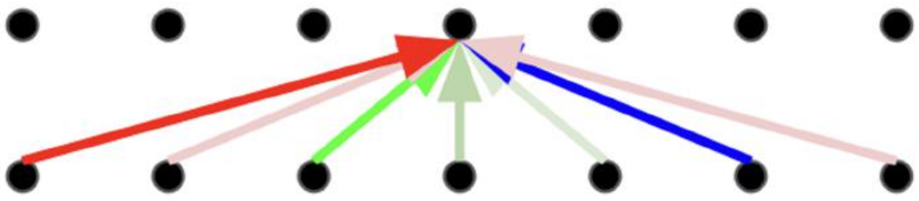


Self-attention when q_i also generated from x

Self-Attention



Convolution

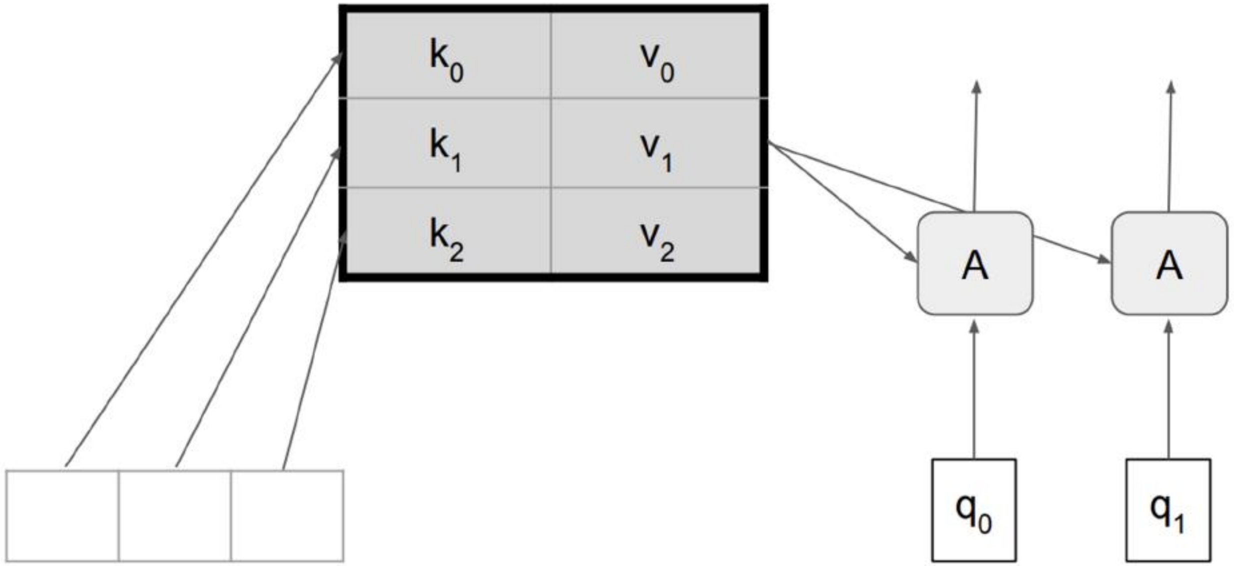


Self-attention

Masked Attention

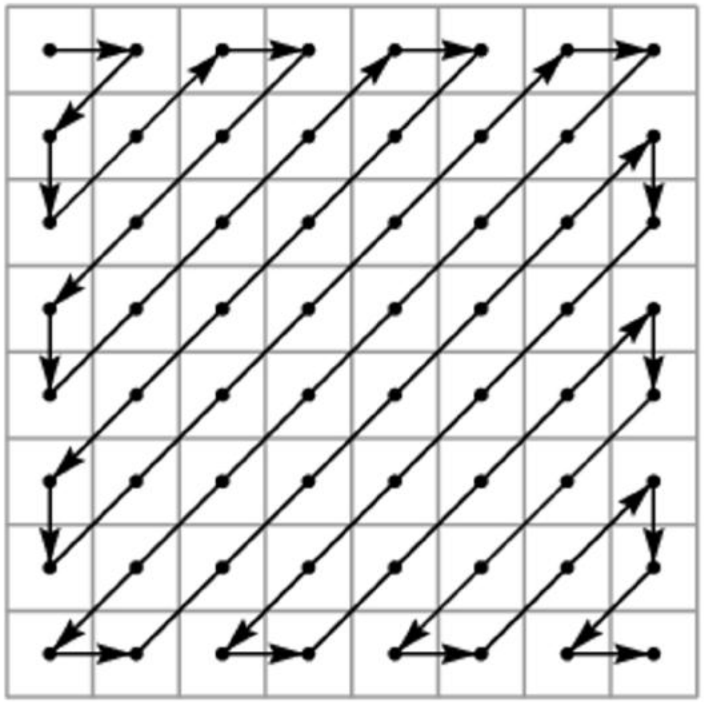
- Dot-Product Attention

$$A(q, K, V) = \sum_i \frac{e^{q \cdot k_i} \text{ - masked}(k_i, q) * 10^{10}}{\sum_j e^{q \cdot k_j} \text{ - masked}(k_i, q) * 10^{10}} v_i$$



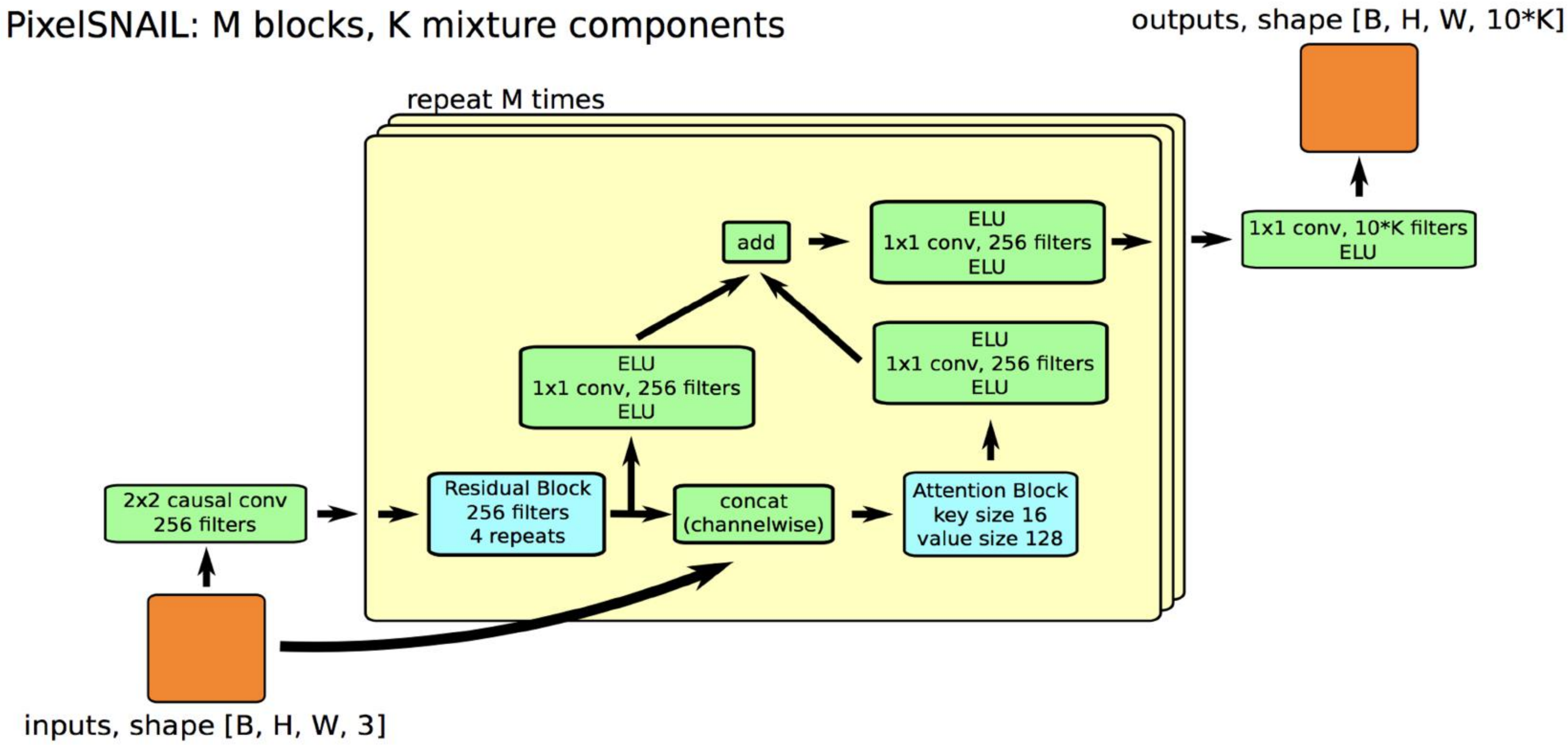
Masked Attention

- Much more flexible than masked convolution. We can design any autoregressive ordering we want
- An example:

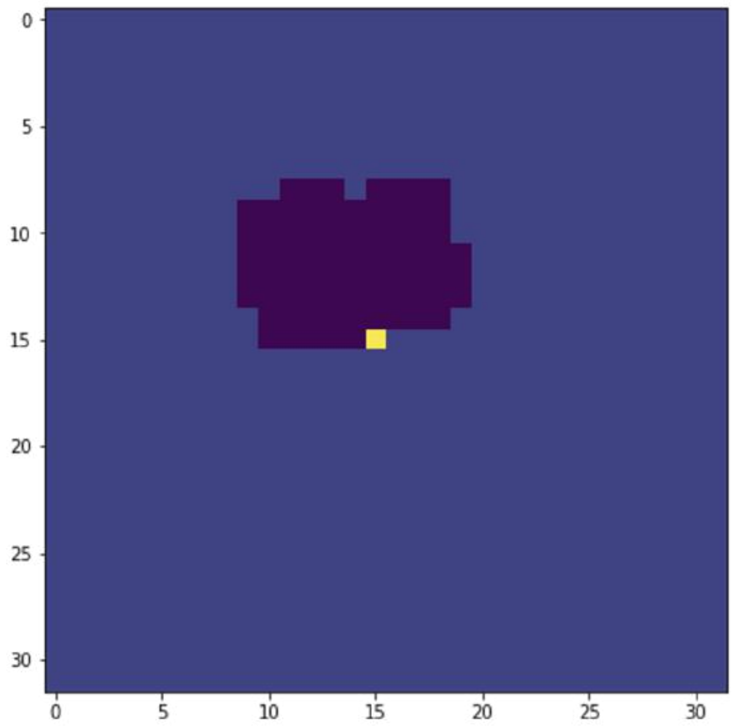


Zigzag ordering
- How to implement with masked conv?
- Trivial to do with masked attention!

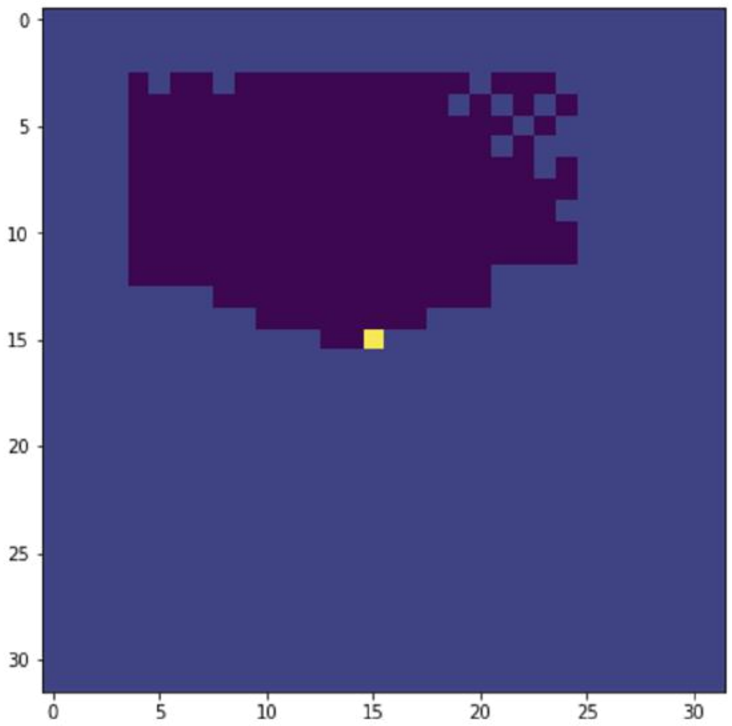
Masked Attention + Convolution



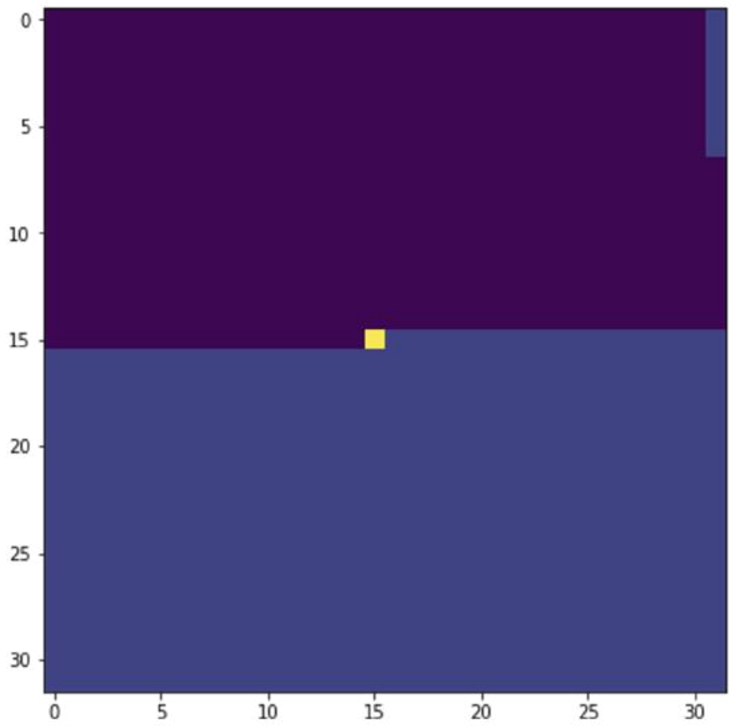
Masked Attention + Convolution



Gated PixelCNN



PixelCNN++



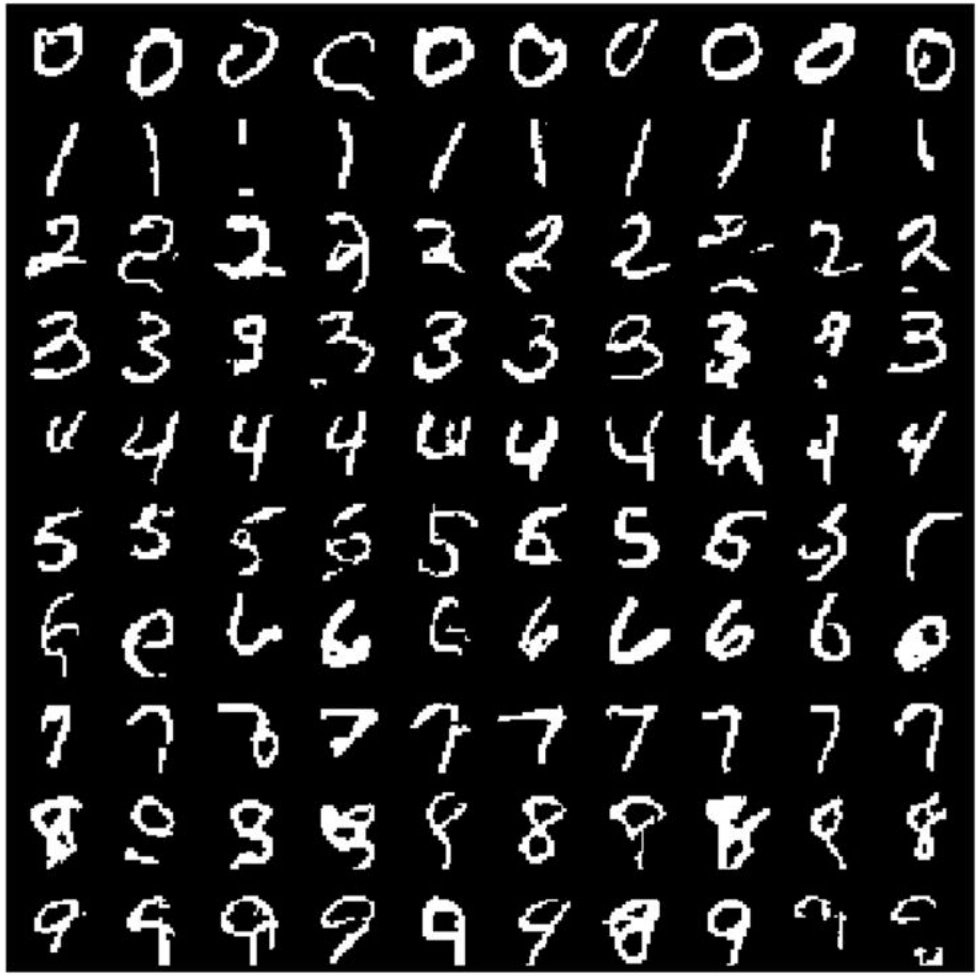
PixelSNAIL

Multi-Head Self-Attention on MNIST

Masked Attention + Convolution

Method	CIFAR-10
Conv DRAW (Gregor et al., 2016)	3.5
Real NVP (Dinh et al., 2016)	3.49
VAE with IAF (Kingma et al., 2016)	3.11
PixelRNN (Oord et al., 2016b)	3.00
Gated PixelCNN (van den Oord et al., 2016b)	3.03
Image Transformer (Anonymous, 2018)	2.98
PixelCNN++ (Salimans et al., 2017)	2.92
Block Sparse PixelCNN++ (OpenAI, 2017)	2.90
PixelSNAIL (ours)	**2.85**

Class-Conditional PixelCNN



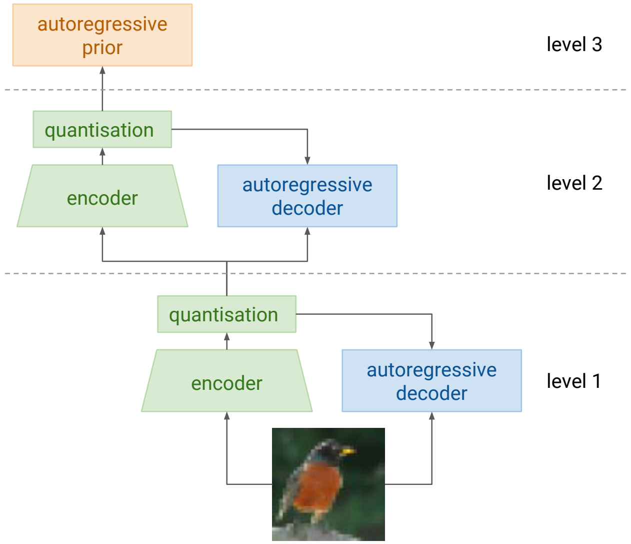
How to condition?

IN: One-hot encoding of the labels

THEN: multiplying by different learned weight matrices in each convolutional layer, and added as a bias channel-wise and broadcasted spatially

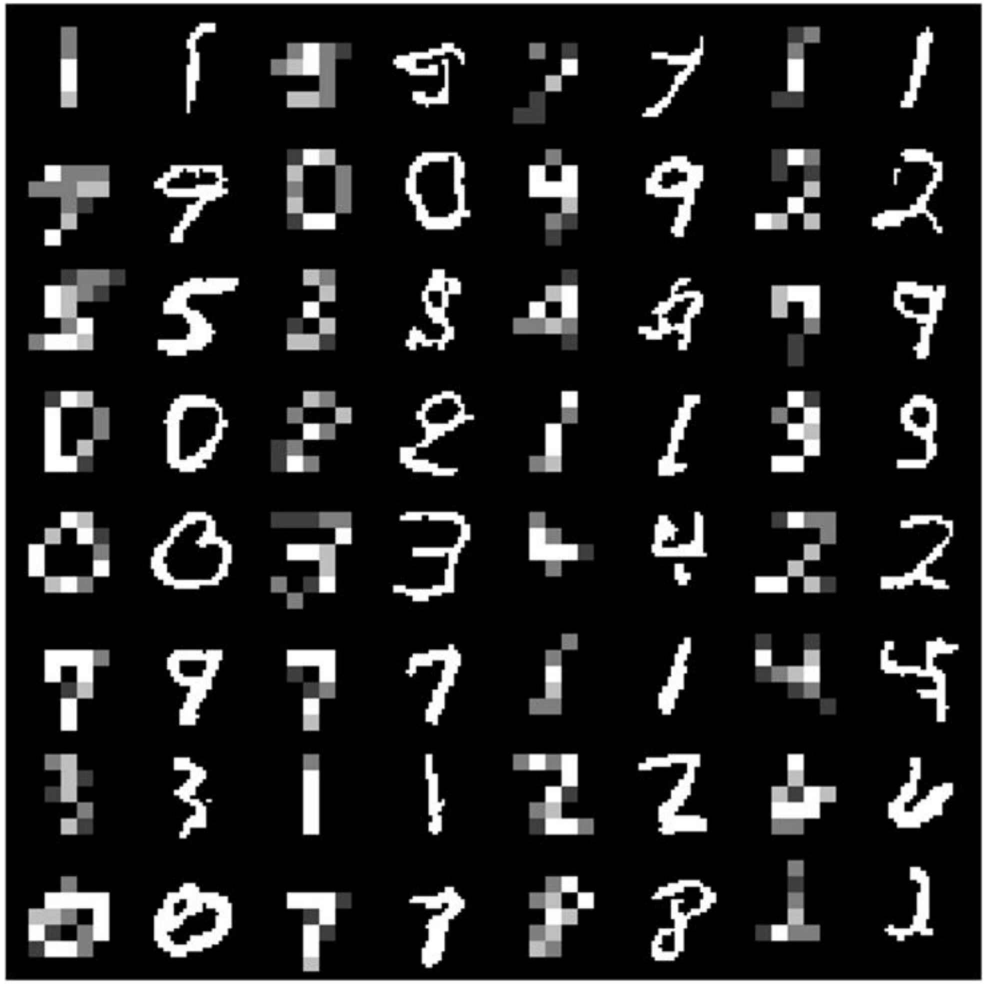
$$\mathbf{y} = \tanh(W_{k,f} * \mathbf{x} + V_{k,f}^T \mathbf{h}) \odot \sigma(W_{k,g} * \mathbf{x} + V_{k,g}^T \mathbf{h})$$

Hierarchical Autoregressive Models with Auxiliary Decoders



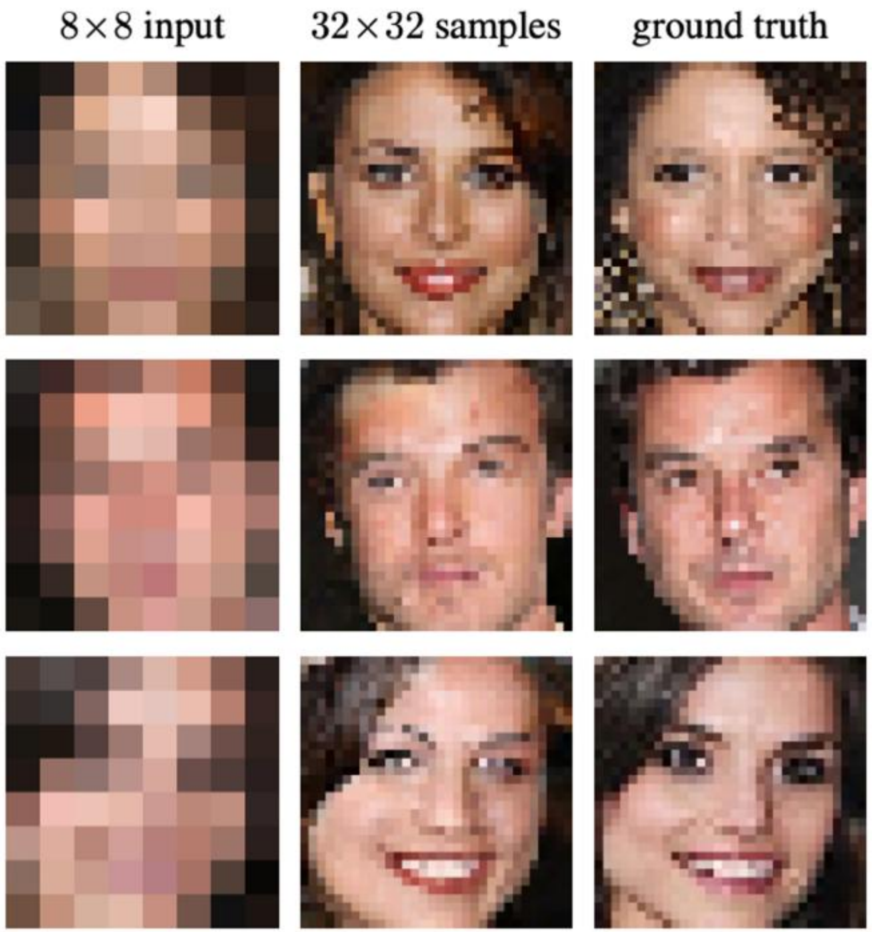
De Fauw, Jeffrey, Sander Dieleman, and Karen Simonyan. "Hierarchical autoregressive image models with auxiliary decoders." arXiv preprint arXiv:1903.04933 (2019).

Image Super-Resolution with PixelCNN



- A PixelCNN is conditioned on 7×7 subsampled MNIST images to generated the corresponding 28×28 image

Pixel Recursive Super Resolution

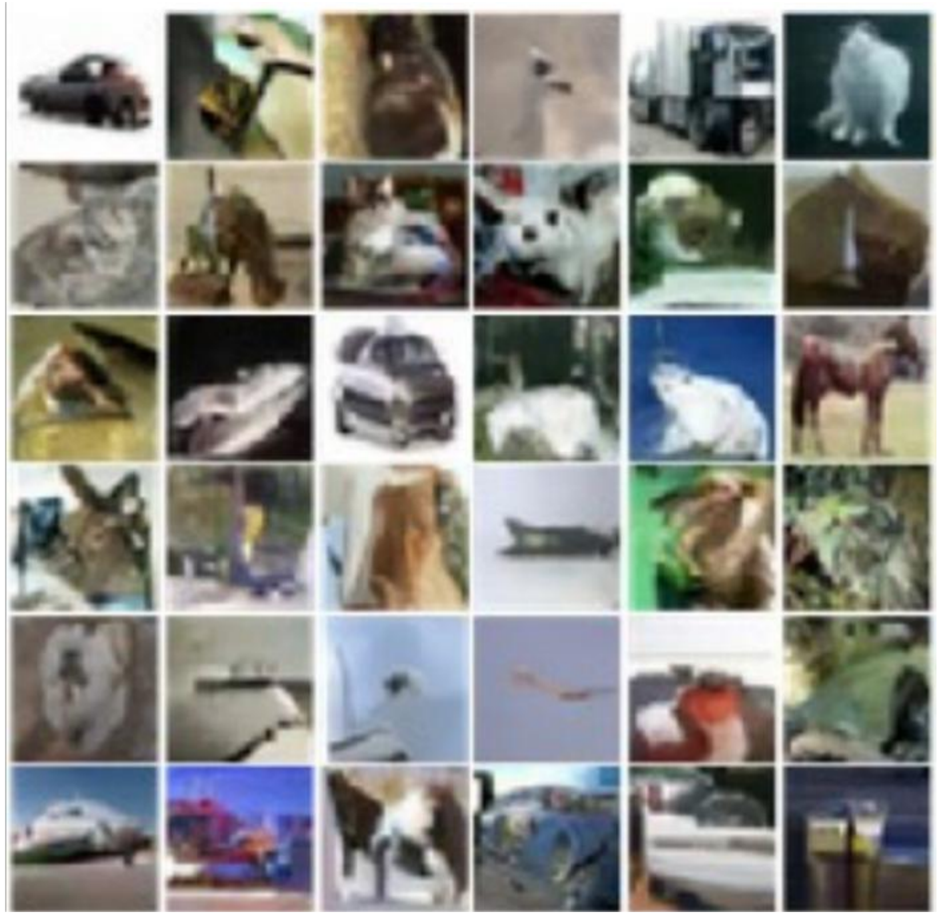


Hierarchy: Grayscale PixelCNN

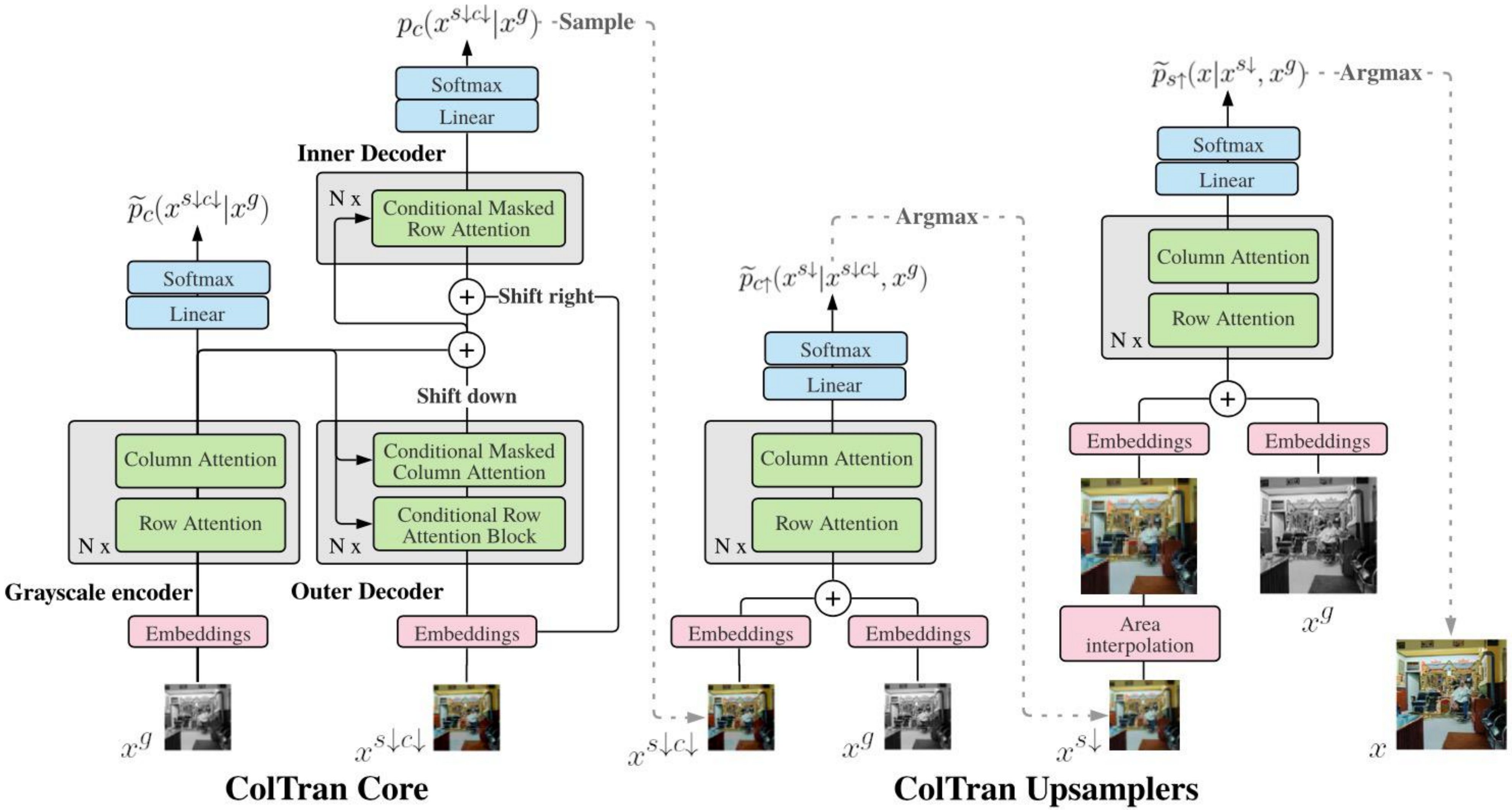
- Design an autoregressive model architecture that takes advantage of the structure of data

- Learn a PixelCNN on binary images, and a PixelCNN conditioned on binary images to generate colored images

PixelCNN Models with Auxiliary Variables for Natural Image Modeling

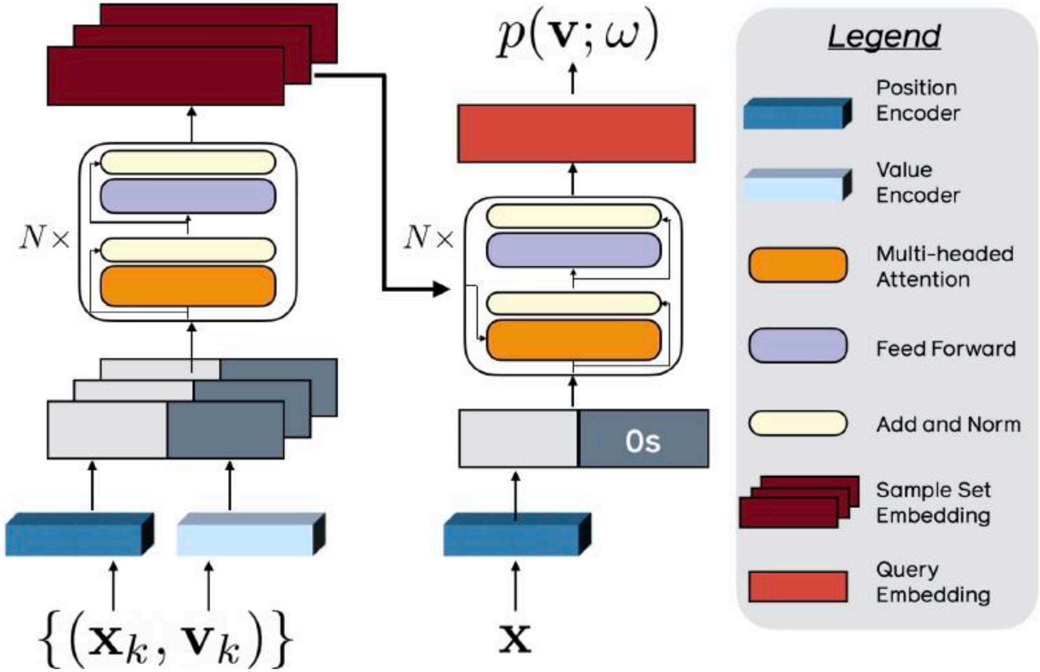


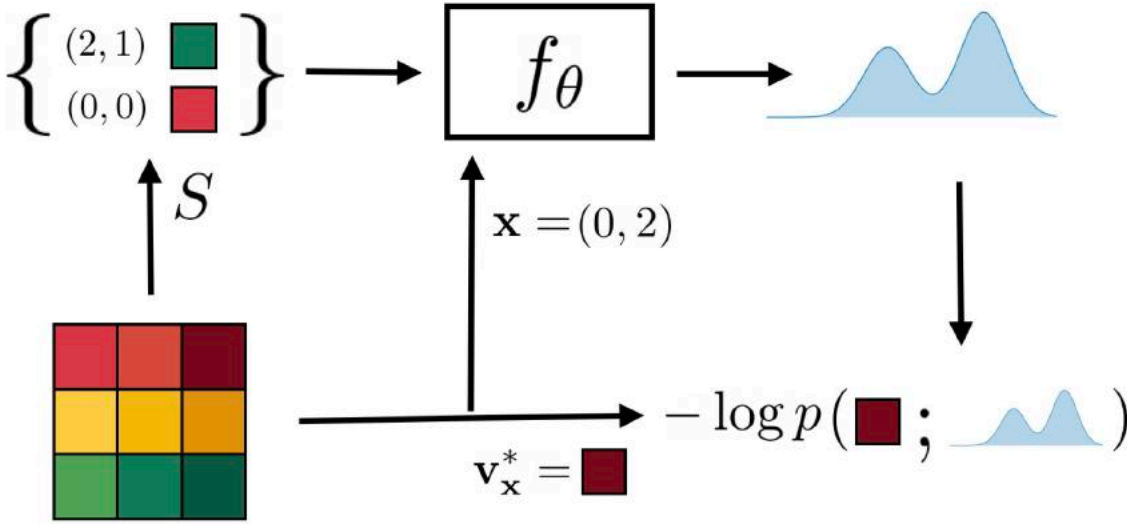
Colorization Transformer



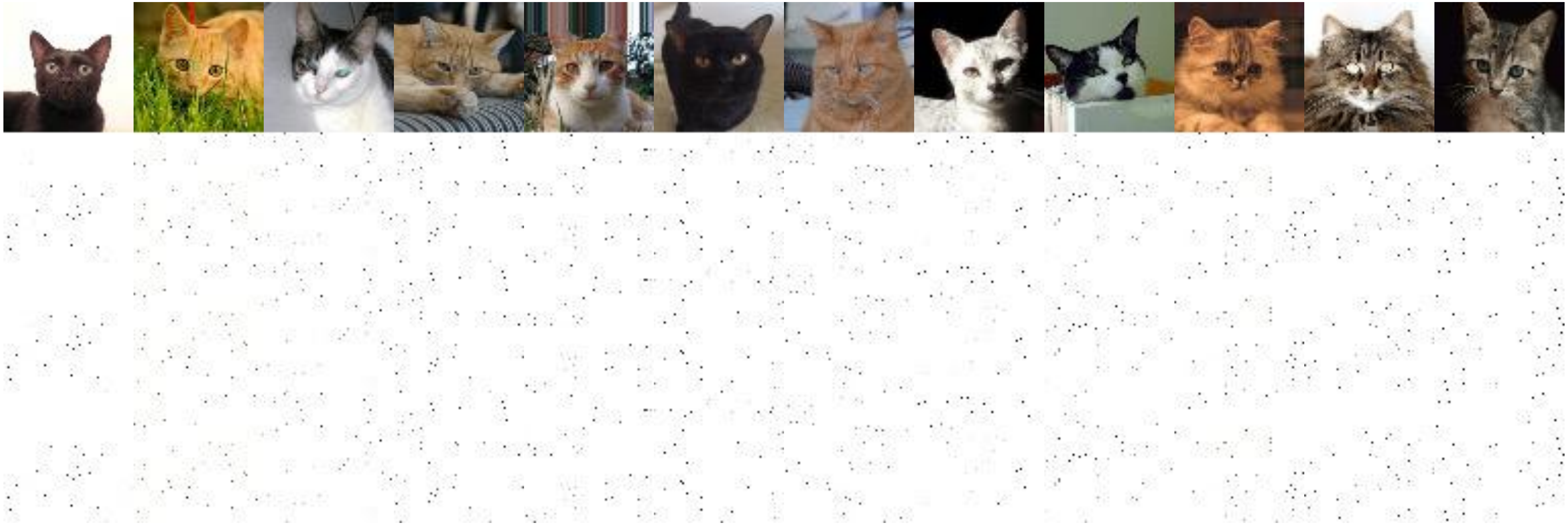
Colorization Transformer

PixelTransformer: Sample Conditioned Signal Generation





PixelTransformer: Sample Conditioned Signal Generation



Top: Ground-truth Image. Bottom: Three random samples generated by our approach given 32 observed pixels (visualized in initial frame of animation).

Neural autoregressive models: The good

Best in class modelling performance:

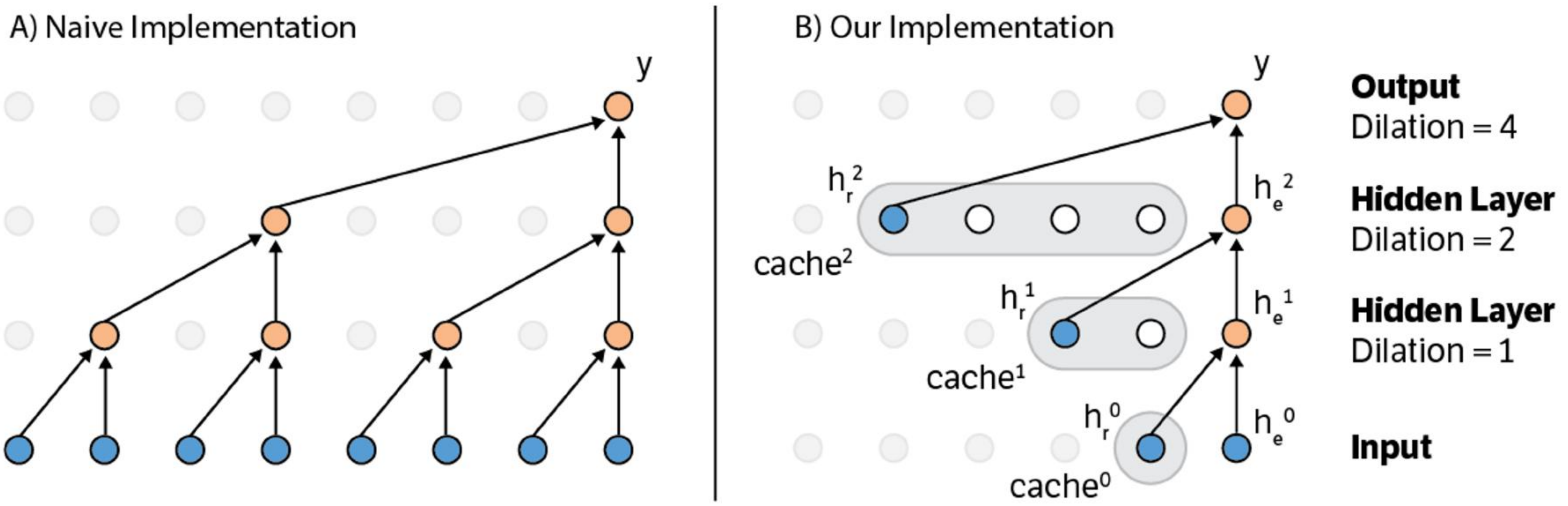
- expressivity - autoregressive factorization is general
- generalization - meaningful parameter sharing has good inductive bias

→ State of the art models on multiple datasets, modalities

Masked autoregressive models: The bad

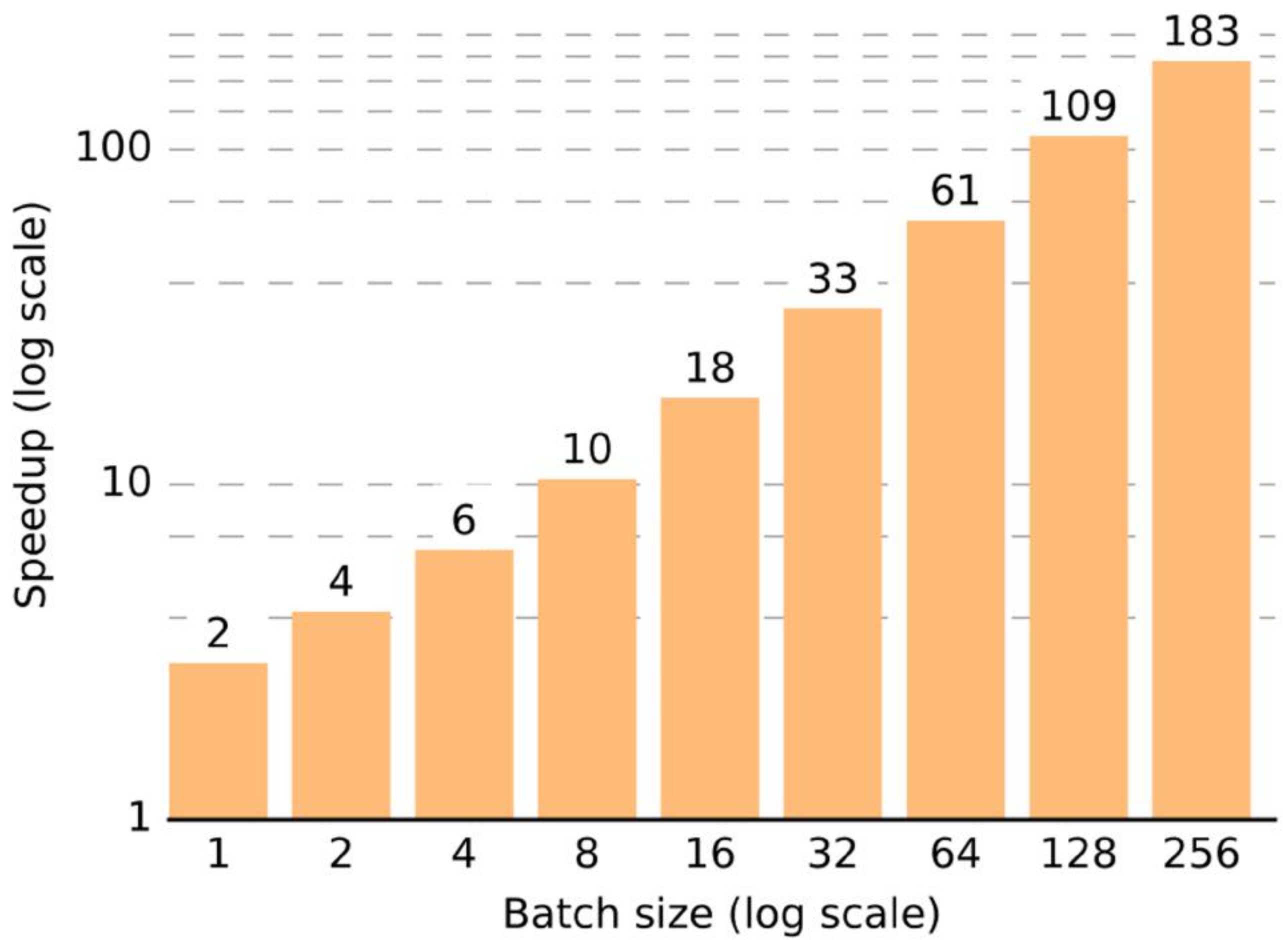
- Sampling each pixel = 1 forward pass!
- 11 minutes to generate 16 32-by-32 images on a Tesla K40 GPU

Speedup by caching activations



https://github.com/PrajitR/fast-pixel-cnn

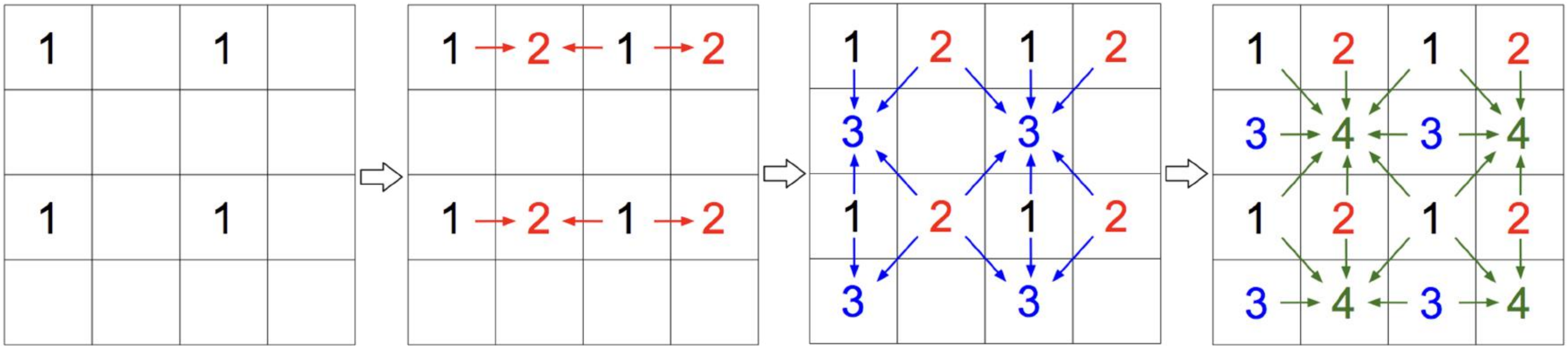
Speedup by caching activations



https://github.com/PrajitR/fast-pixel-cnn

Speedup by breaking autoregressive pattern

- O(d) → O(log(d)) by parallelizing within groups {2, 3, 4}
- Cannot capture dependencies within each group: this is a fine assumption if all pixels in one group are conditionally independent
  - Most often they are not, then you trade expressivity for sampling speed



Multiscale PixelCNN

Model	scale	time	speedup
$O(N)$ PixelCNN	32	120.0	$1.0\times$
$O(\log N)$ PixelCNN	32	1.17	$102\times$
$O(\log N)$ PixelCNN, in-graph	32	1.14	$105\times$

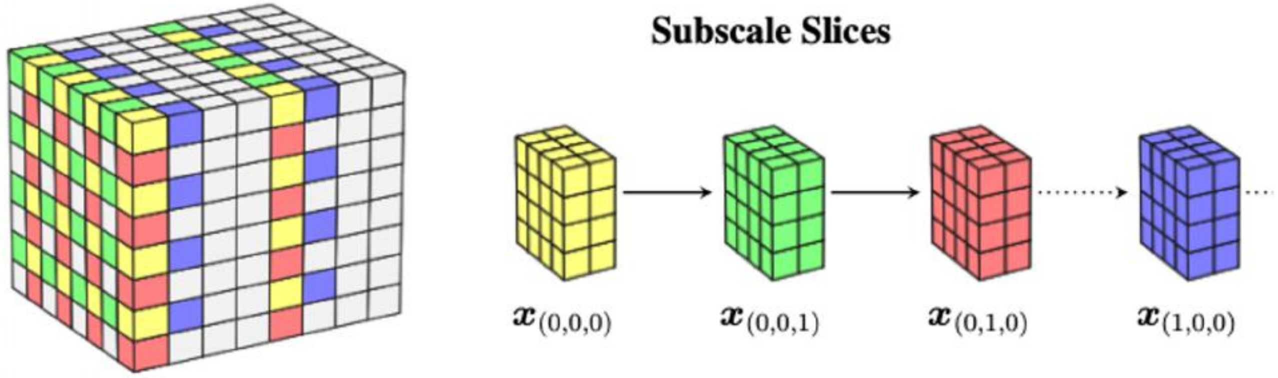
Improved sampling speed

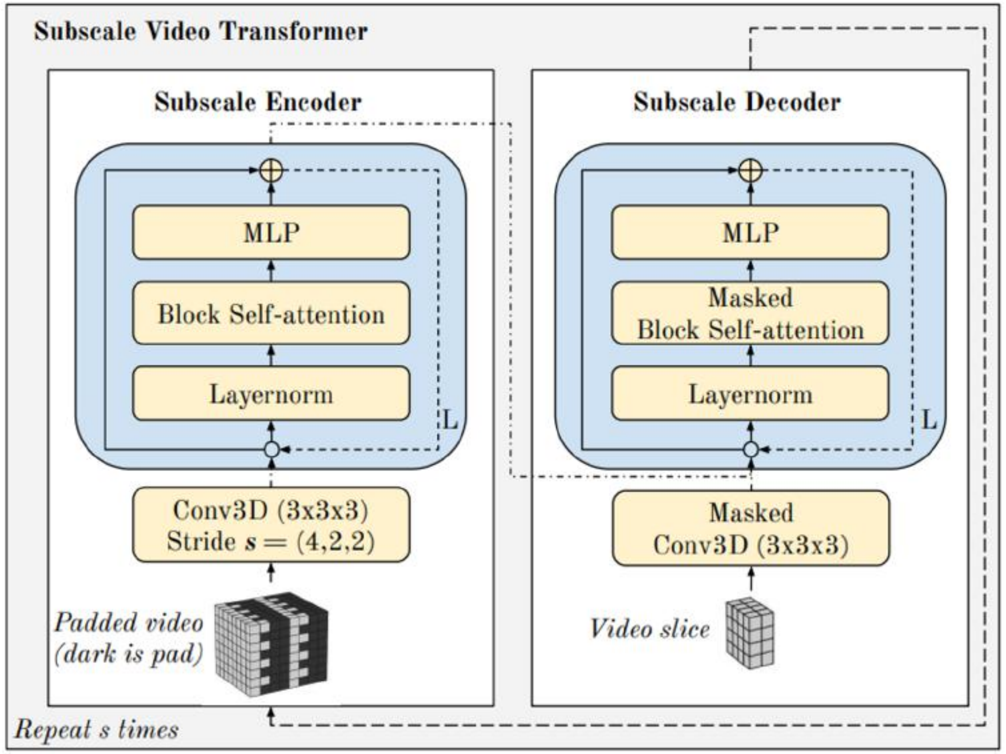
Model	32	64	128
PixelRNN	3.86 (3.83)	3.64(3.57)	-
PixelCNN	3.83 (3.77)	3.57(3.48)	-
Real NVP	4.28(4.26)	3.98(3.75)	-
Conv. DRAW	4.40(4.35)	4.10(4.04)	-
Ours	3.95(3.92)	3.70(3.67)	3.55(3.42)

Table 3. ImageNet negative log-likelihood in bits per sub-pixel at 32×32, 64×64 and 128×128 resolution.

More limited modelling capacity

Scaling Autoregressive Video Models



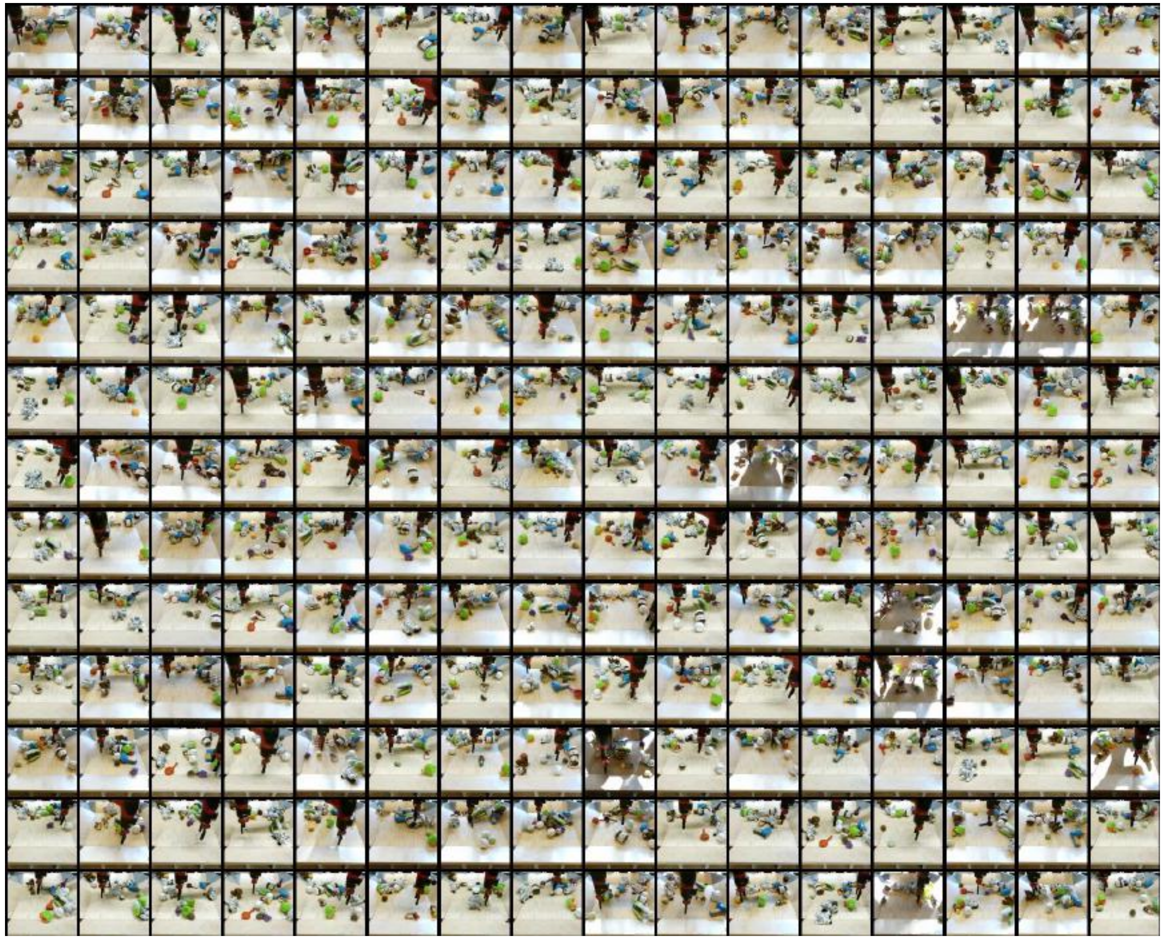


[Dirk Weissenborn, Oscar Tackstrom, Jakob Uszkoreit. "Scaling Autoregressive Video Models." arXiv 1906.02634 (2019)]

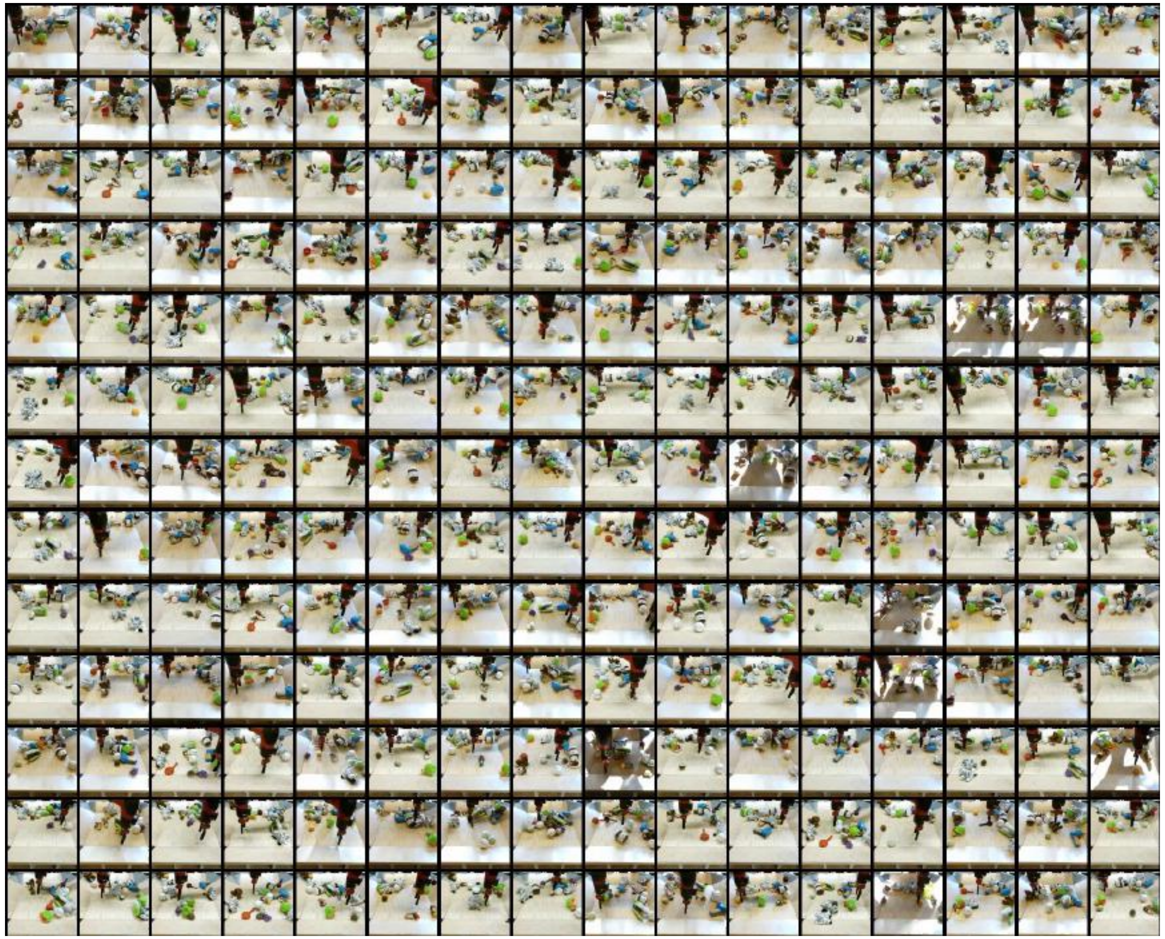
Scaling Autoregressive Video Models

BAIR Robot Pushing

Large Spatiotemporal Subscaling



Small Spatiotemporal Subscaling

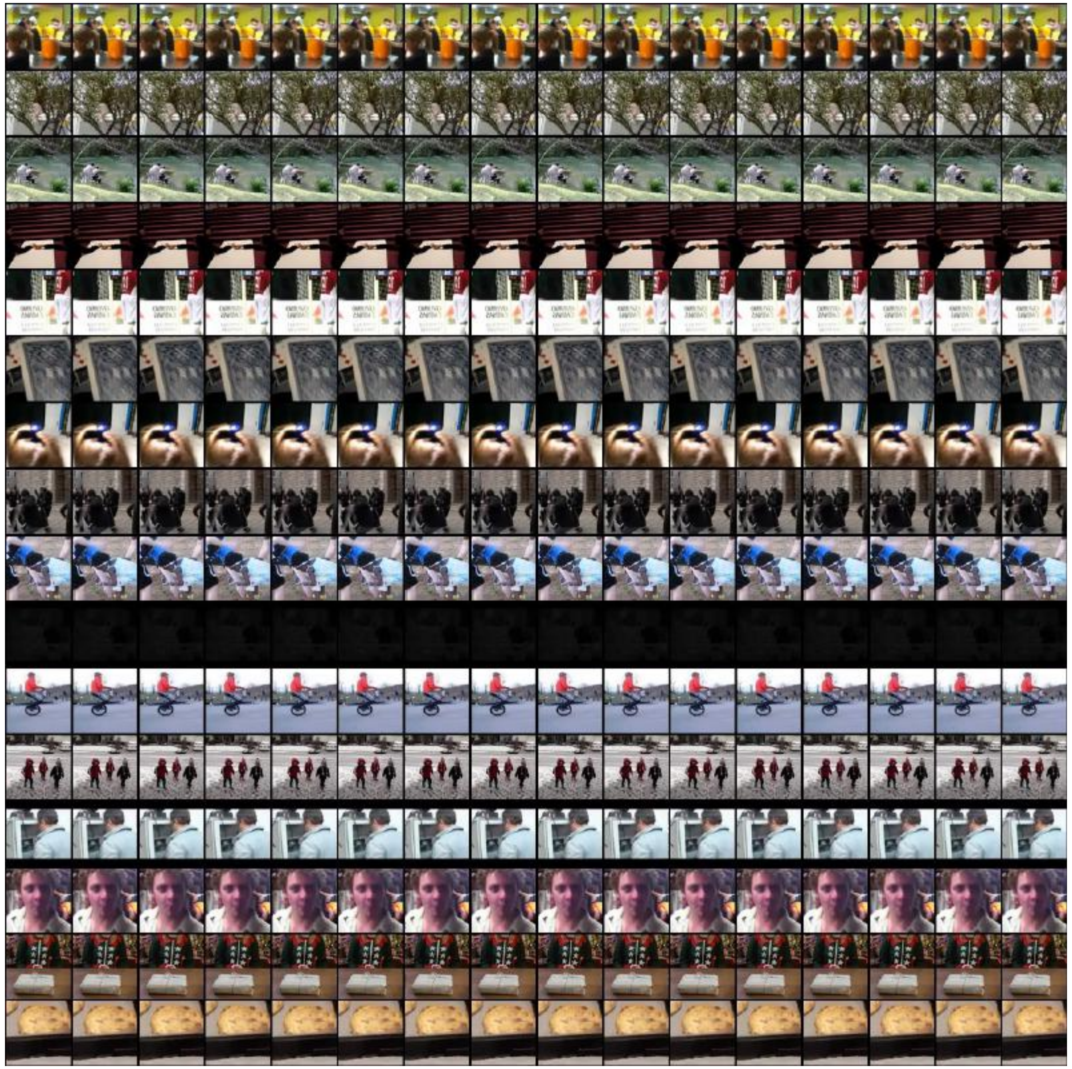


[Dirk Weissenborn, Oscar Tackstrom, Jakob Uszkoreit. "Scaling Autoregressive Video Models." arXiv 1906.02634 (2019)]

Scaling Autoregressive Video Models Kinetics

Cooking (left-to-right by likelihood)

Full Kinetics (left-to-right by likelihood)



[Dirk Weissenborn, Oscar Tackstrom, Jakob Uszkoreit. "Scaling Autoregressive Video Models." arXiv 1906.02634 (2019)]

Natural Image Manipulation for Autoregressive Models using Fisher Scores

- Main challenge:
  - How to get a latent representation from PixelCNN?
  - Why hard? The random input happens on a per-pixel sample basis

- Proposed solution
  - Use Fisher score

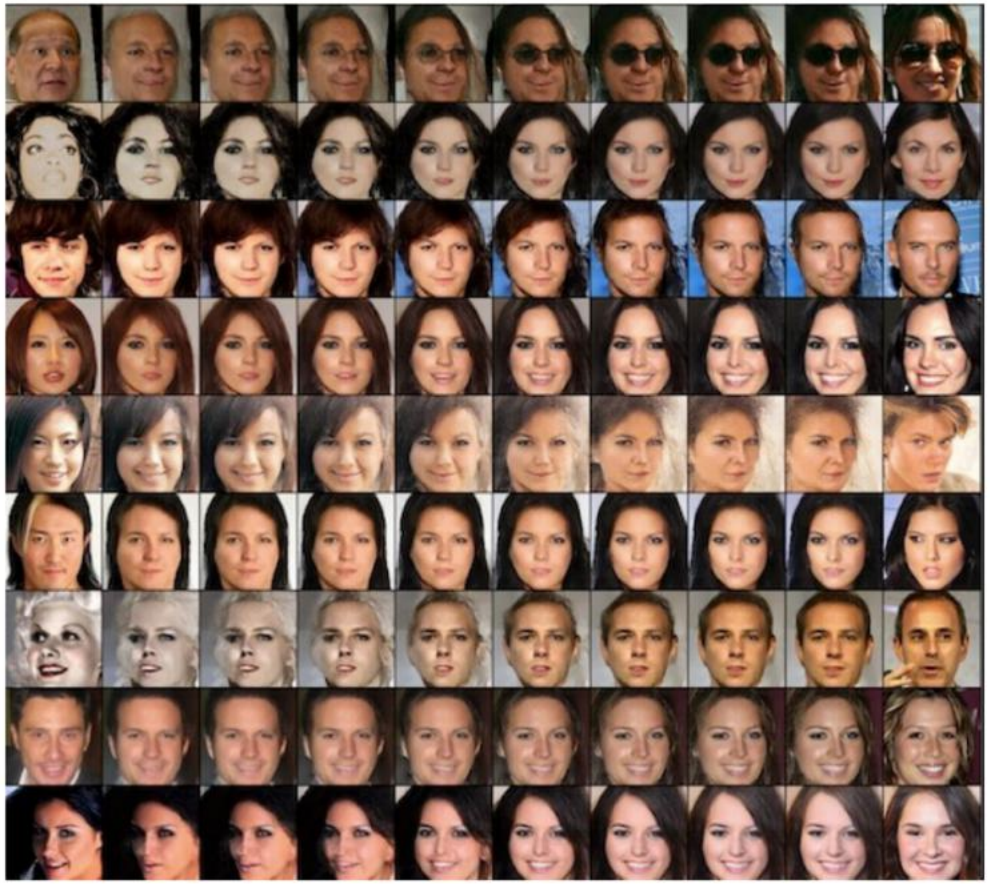
$$\dot{\ell}(x;\theta) = \nabla_\theta \log p_\theta(x)$$

Note: applicable to any likelihood model

[Wilson Yan, Jonatha Ho, Pieter Abbeel. "Natural Image Manipulation for Autoregressive Models using Fisher Scores." arXiv 1912.05015

Natural Image Manipulation for Autoregressive Models using Fisher Scores

(c) Activations (Interpolation)



(d) Fisher score (Interpolation)

[Wilson Yan, Jonatha Ho, Pieter Abbeel. "Natural Image Manipulation for Autoregressive Models using Fisher Scores." arXiv 1912.05015

Natural Image Manipulation for Autoregressive Models using Fisher Scores

[Wilson Yan, Jonatha Ho, Pieter Abbeel. “Natural Image Manipulation for Autoregressive Models using Fisher Scores.” arXiv 1912.05015

Next lecture:
Flow-Based Models