Shape Scale: Representing Shapes at Their Absolute Scales

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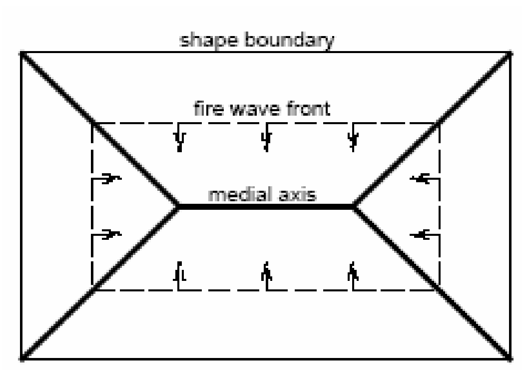
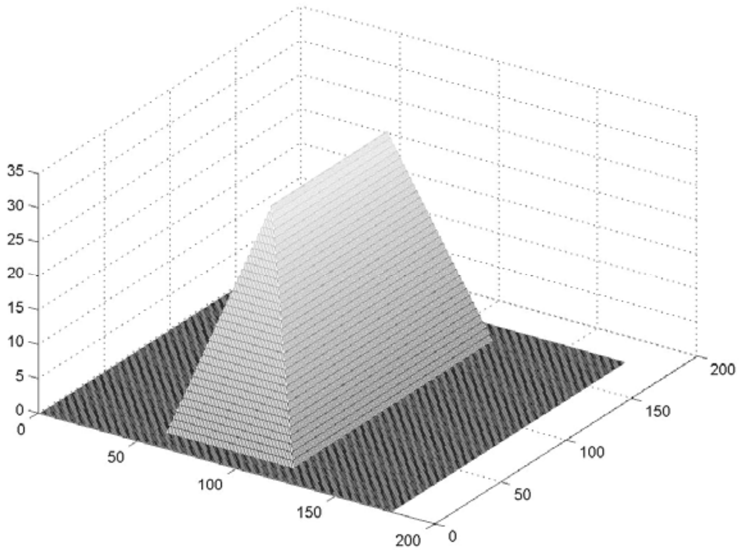
Blum's Morphological Skeleton

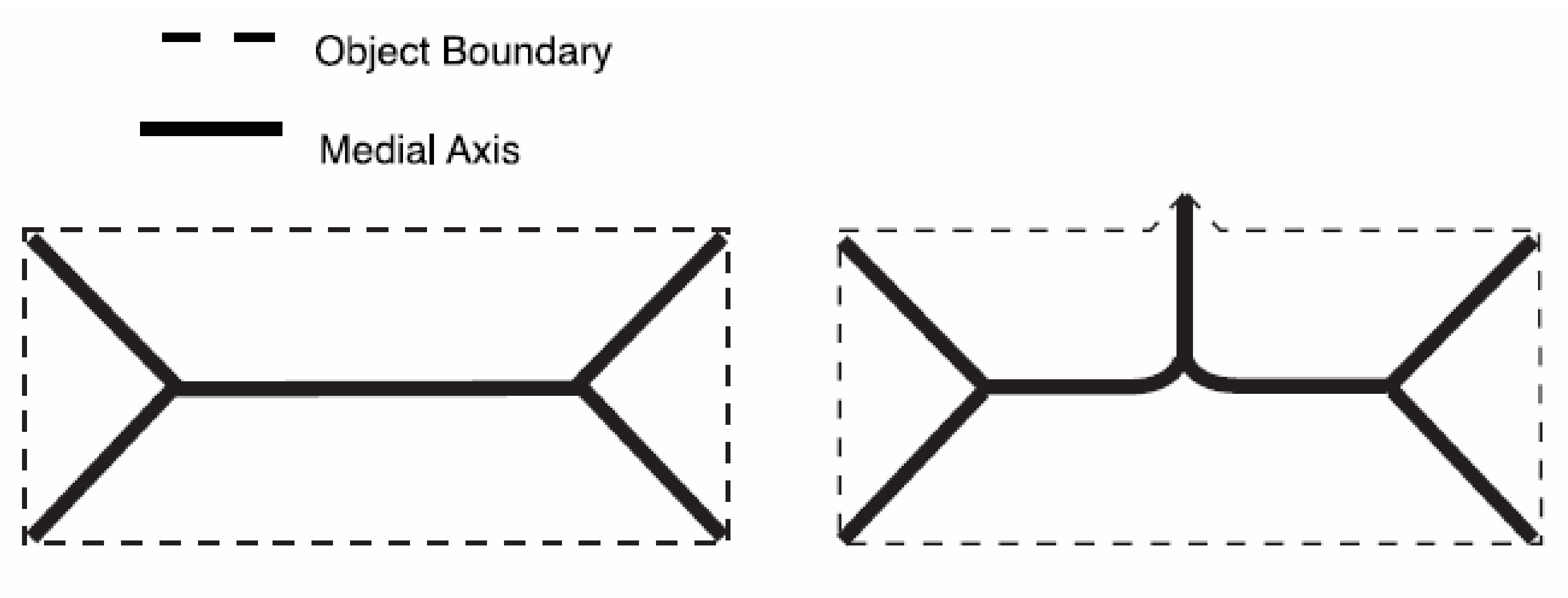
- Evolving fire front analogy





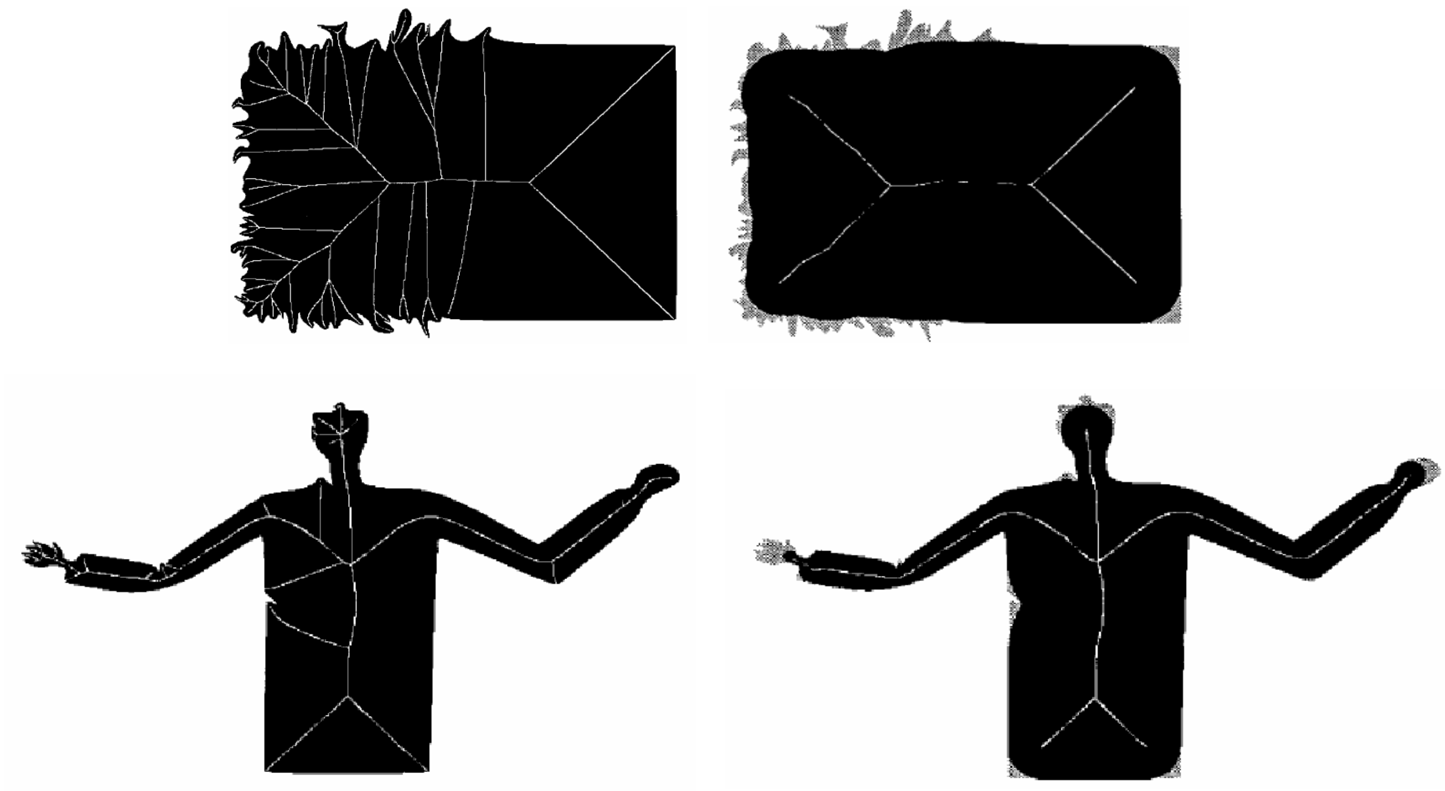
$$\frac{\partial C}{\partial t} = 1.\vec{N}$$

Instability of Minor Perturbations



Blum’s morphological skeleton is instable

Pruning Morphological Skeleton

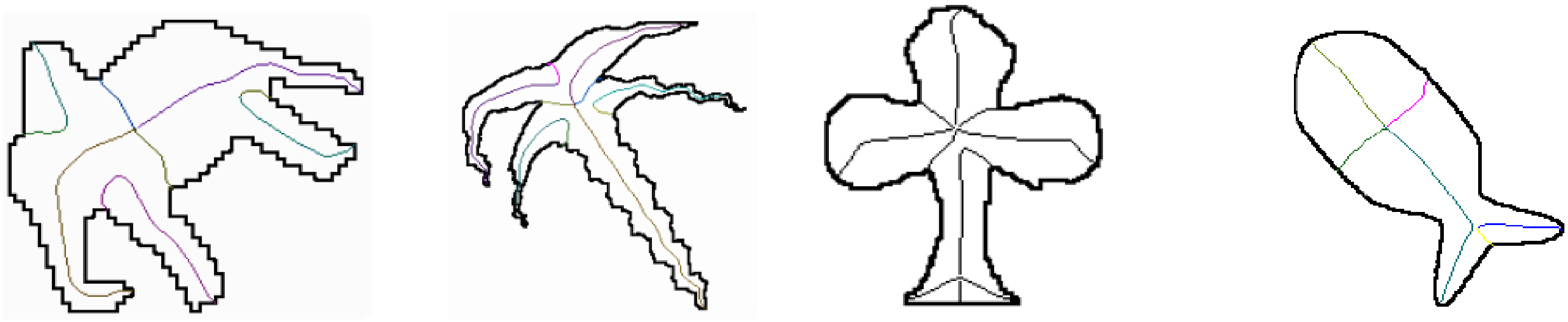


"*Pruning Medial Axes*",
D. Shaked and A. M. Bruckstein, CVIU, 1998

Disconnected Skeleton

(Preliminary Work at ICCV'05)

- Compute the local symmetries only at the locations where it can be accurately determined.
- Select a shape dependent scale σ^* in which its representation is most stable.



Disconnected Skeleton

- Construct the distance surface *ø* as the solution of the following linear diffusion equation at a special scale *σ**:

$$\frac{\partial}{\partial\sigma}\phi(x,y,\sigma) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\phi(x,y,\sigma) \qquad (1)$$

$$\phi(x,y,\sigma)|_{(x,y)\in\Gamma} = 1$$

- Related to the edge strength function *v* used in Tari, Shah and Pien, CVIU, 1997

The Edge Strength Function

- Solution of the following PDE:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)v(x,y)-\frac{v(x,y)}{\rho^2}=0 \qquad (2)$$

$$v(x,y)|_{(x,y)\in\Gamma}=1$$

equivalently:

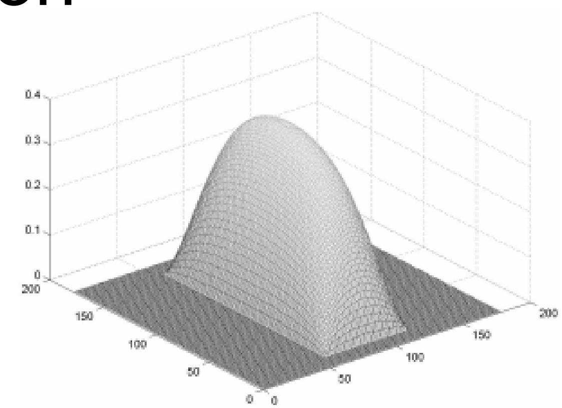
$$\frac{\partial}{\partial\sigma}v(x,y,\sigma)=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)v(x,y,\sigma)-\frac{v(x,y,\sigma)}{\rho^2} \qquad (3)$$

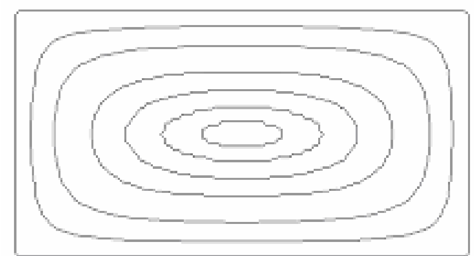
$$v(x,y)|_{(x,y)\in\Gamma}=1$$

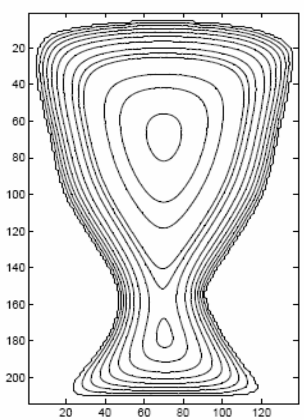
- As $\rho\rightarrow 0$ v function becomes an approximation to the discontinuity locus of the Mumford-Shah segmentation model.

The Edge Strength Function

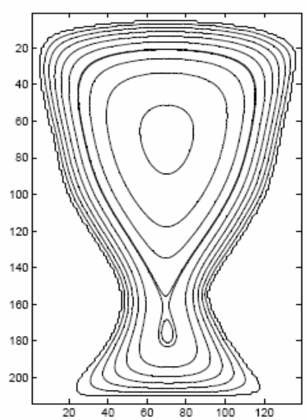
- For larger values of ρ v function acts like a level set function



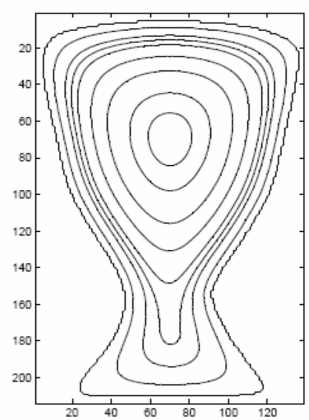




$\rho = 12$



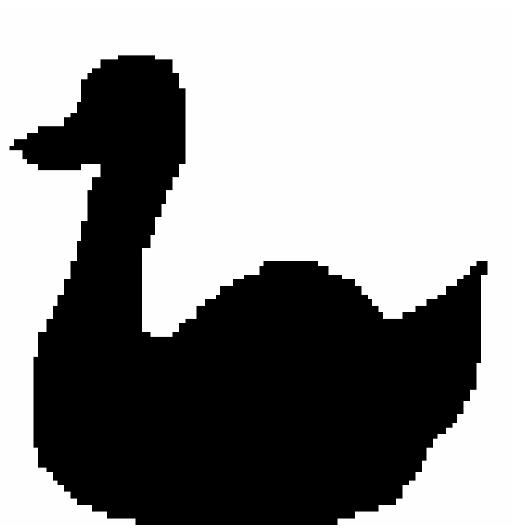
$\rho = 16$

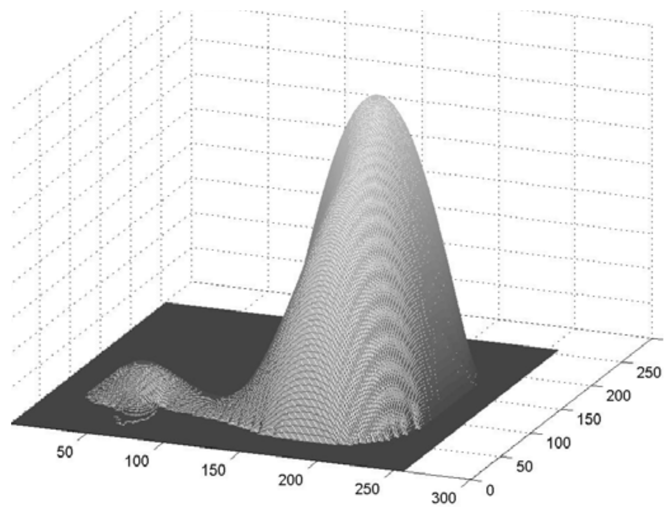


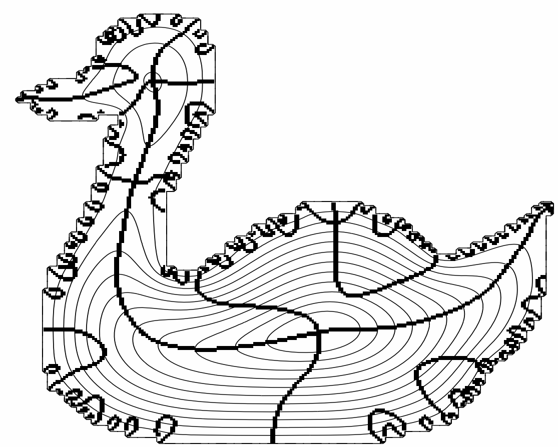
$\rho = 32$

Symmetry Point Computation

- $\kappa \; \alpha \; |\nabla v|$
- Zero crossings of $\frac{d|\nabla v|}{ds}$

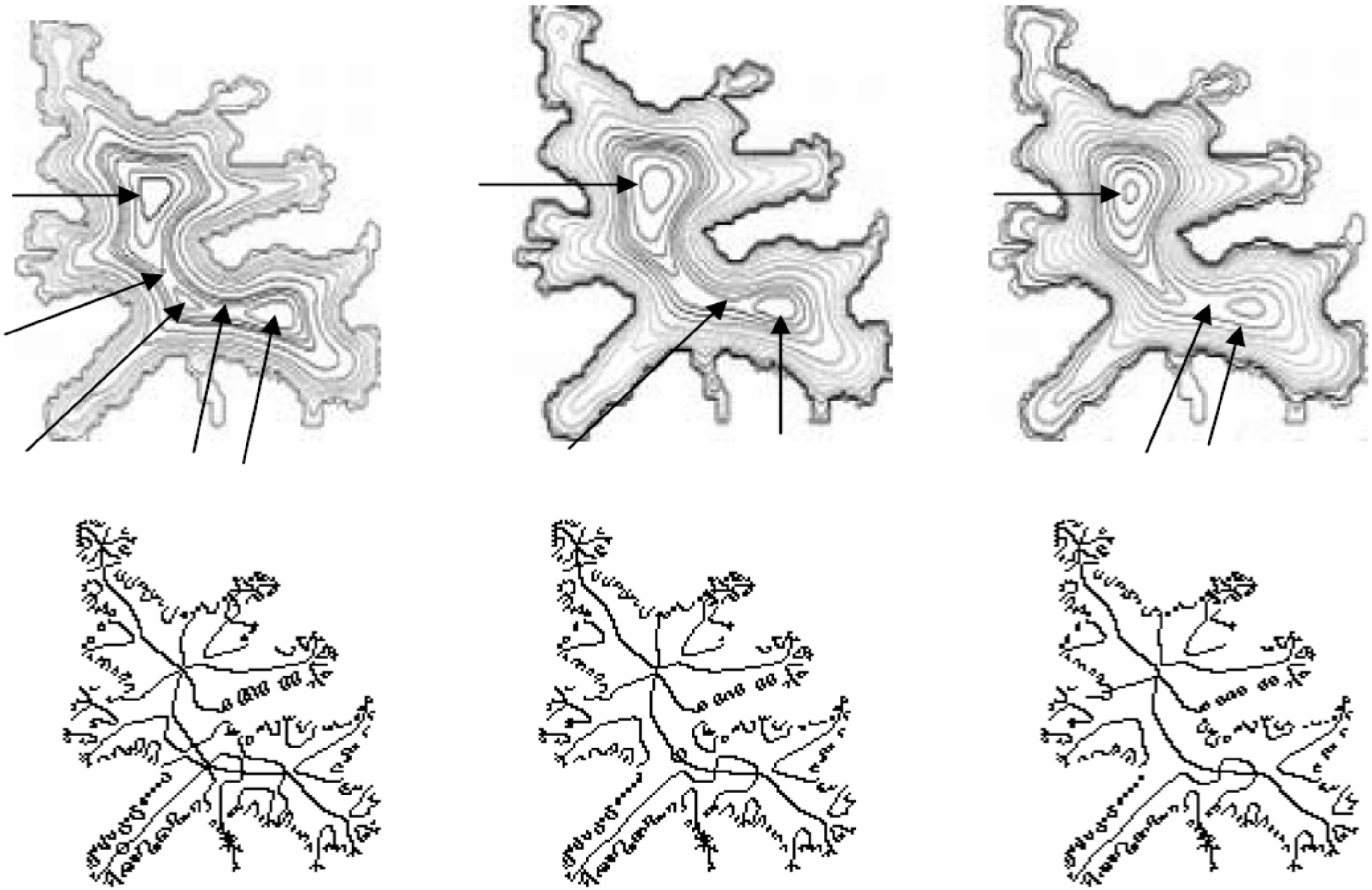






$$\frac{d|\nabla v|}{ds} = \frac{((v_y^2 - v_x^2)v_{xy} - v_x v_y (v_{yy} - v_{xx}))}{|\nabla v|^2}$$

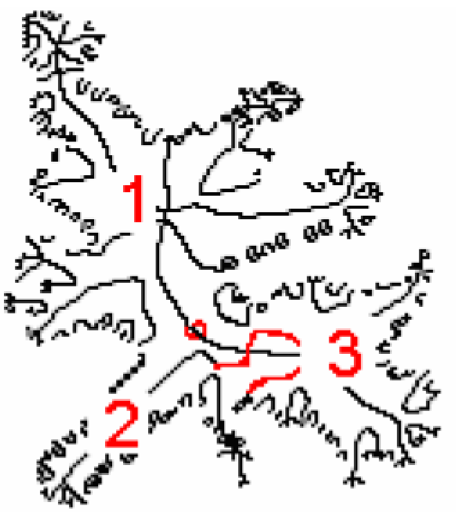
Saddle Point Instability



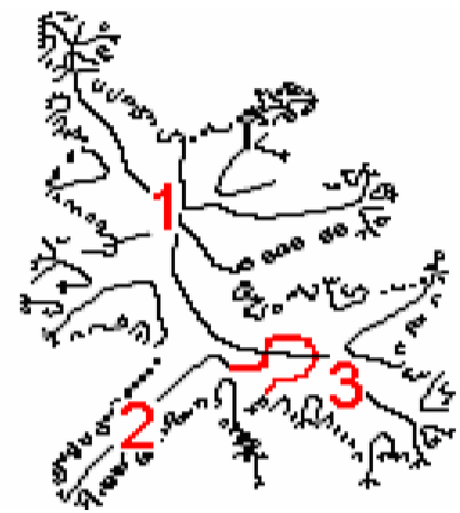
$\rho = 4$ $\rho = 8$ $\rho = 16$

Saddle Point Instability

- Spurious symmetry points may arise due to insufficient diffusion



$\rho = 8$



$\rho = 16$

New Surface Computation

- Diffuse the surface until a single extremum exists
- As $\rho \rightarrow \infty$ edge strength function approaches to the steady state solution of (1):

$$\frac{\partial}{\partial \sigma} v(x, y, \sigma) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) v(x, y, \sigma) - \frac{v(x, y, \sigma)}{\rho^2} \qquad (3)$$

$$v(x, y)\big|_{(x,y)\in\Gamma} = 1$$

$$\frac{\partial}{\partial \sigma} \phi(x, y, \sigma) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \phi(x, y, \sigma) \qquad (1)$$

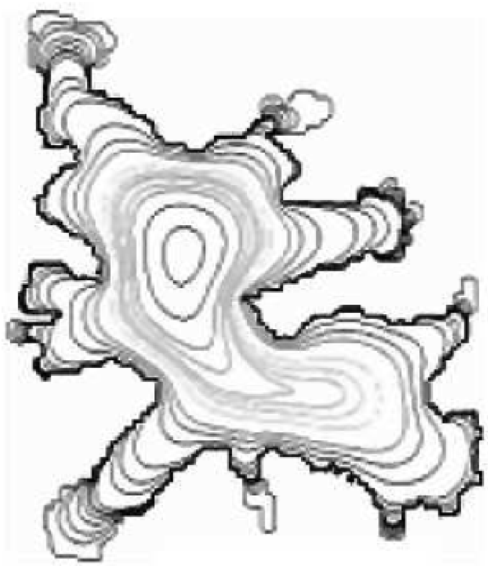
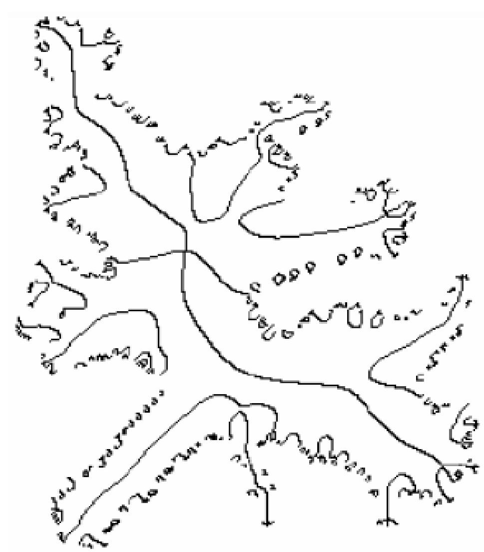
$$\phi(x, y, \sigma)\big|_{(x,y)\in\Gamma} = 1$$

- Steady state solution is identically equal to 1 which does not contain any shape information.

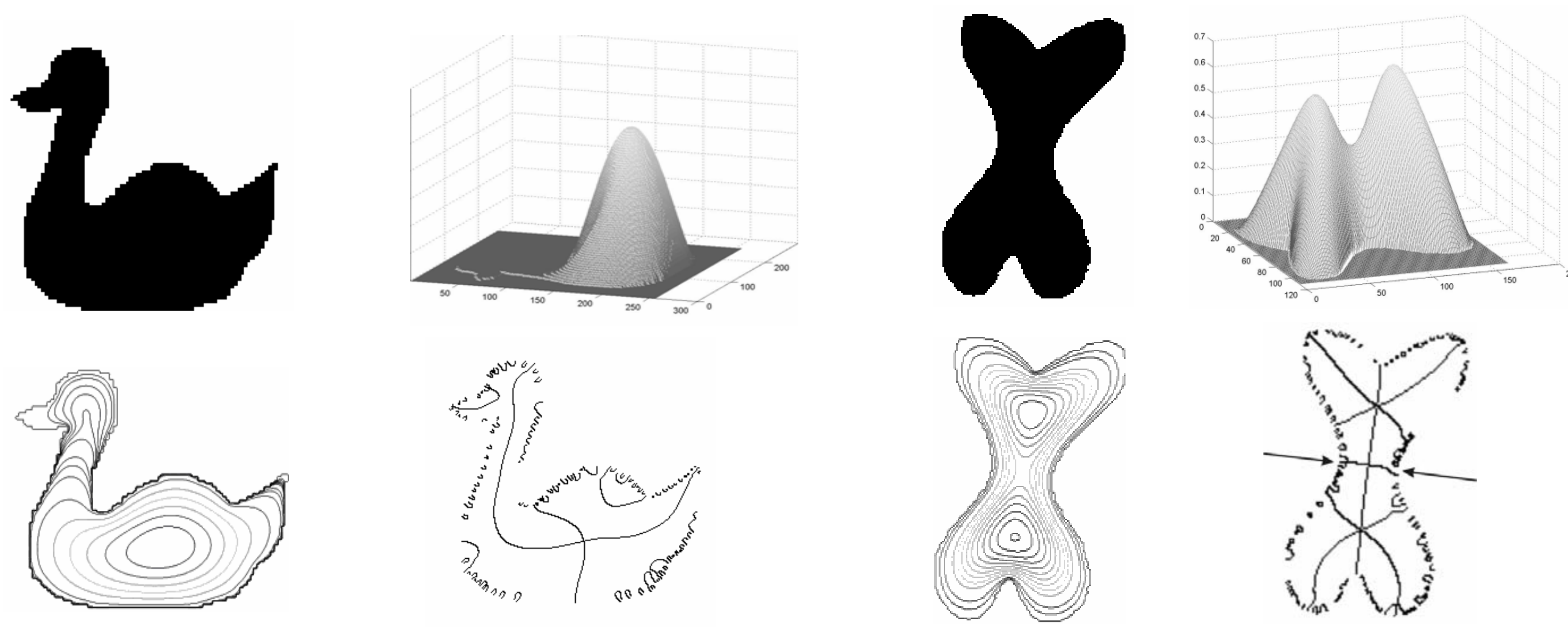
New Surface Computation

- Select a large amount of diffusion σ^* that will not produce a completely flat surface.

- Value of σ^* depends on the specific shape

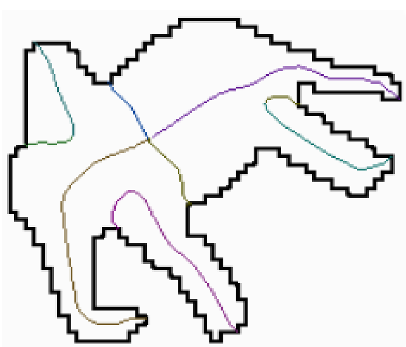
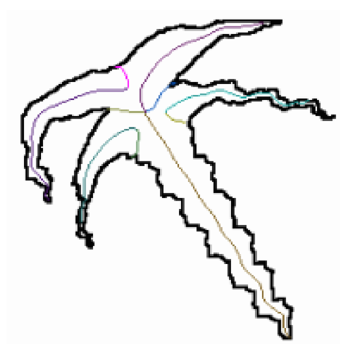
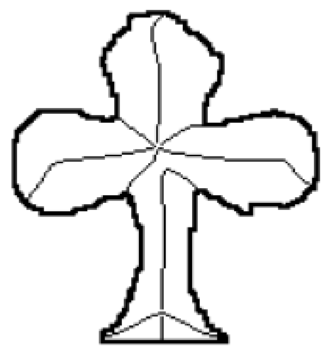
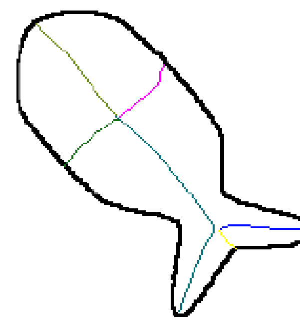



From ∅ To Symmetry Axes

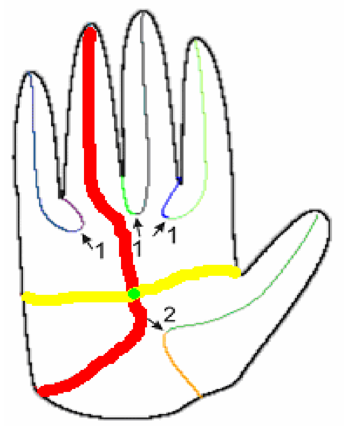


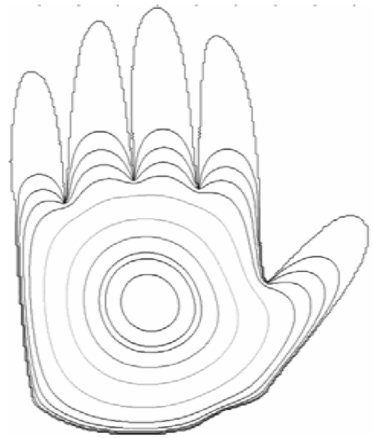
From ∅ To Symmetry Axes

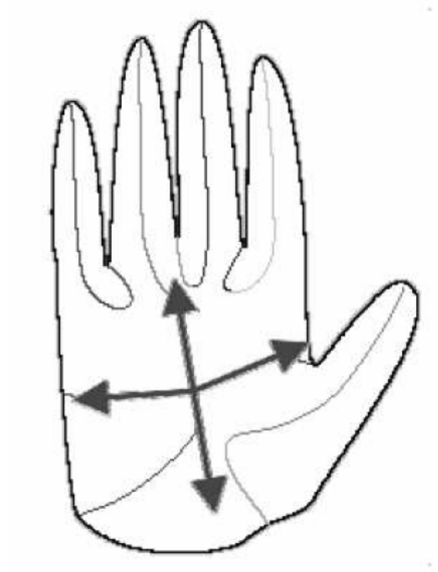
- Each primitive is a local symmetry branch
  - starting at a local curvature extremum of the boundary and
  - ending at a disconnection point

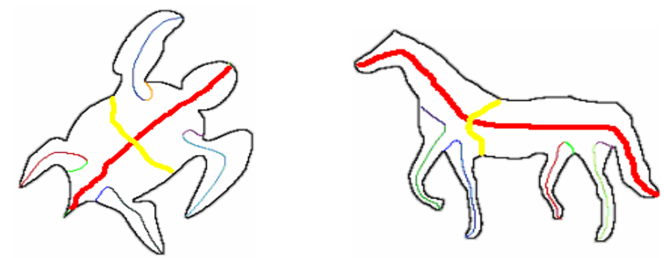
   

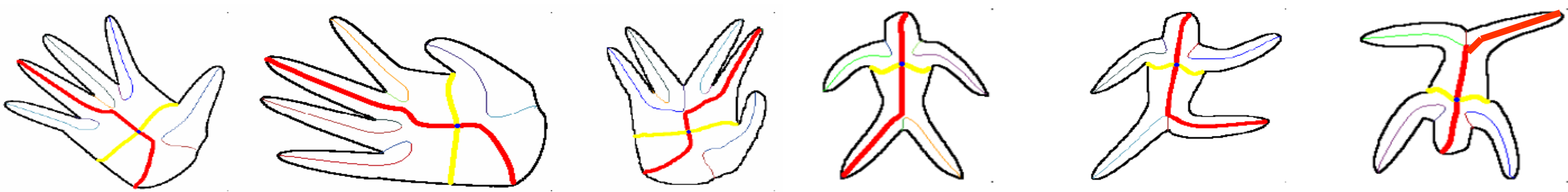
Global Coordinate Frame





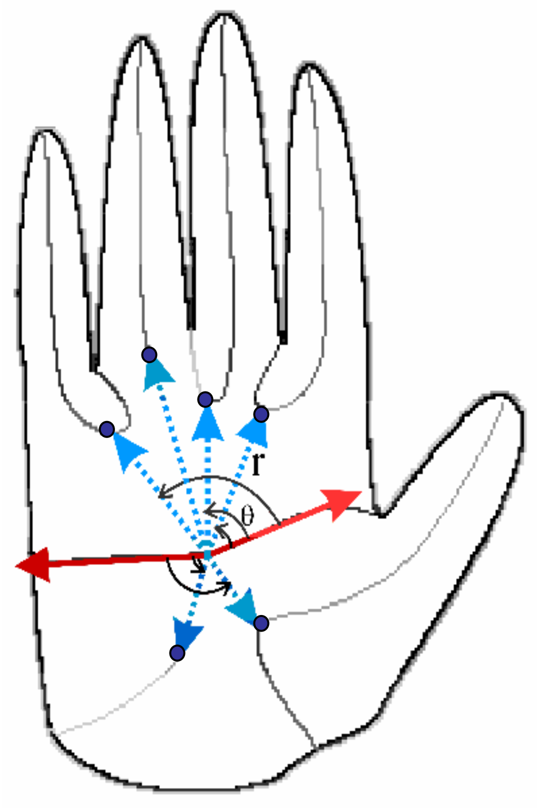


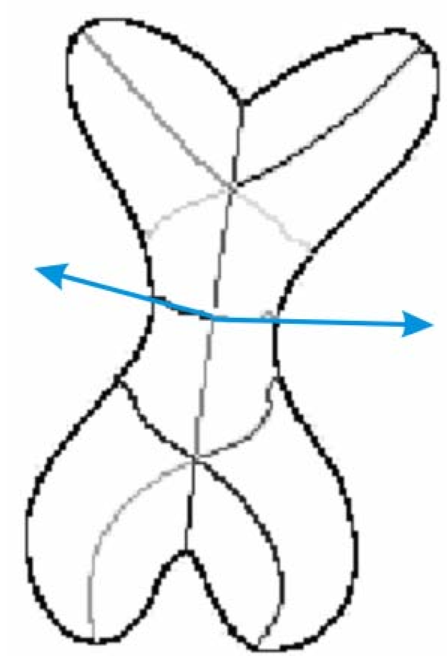


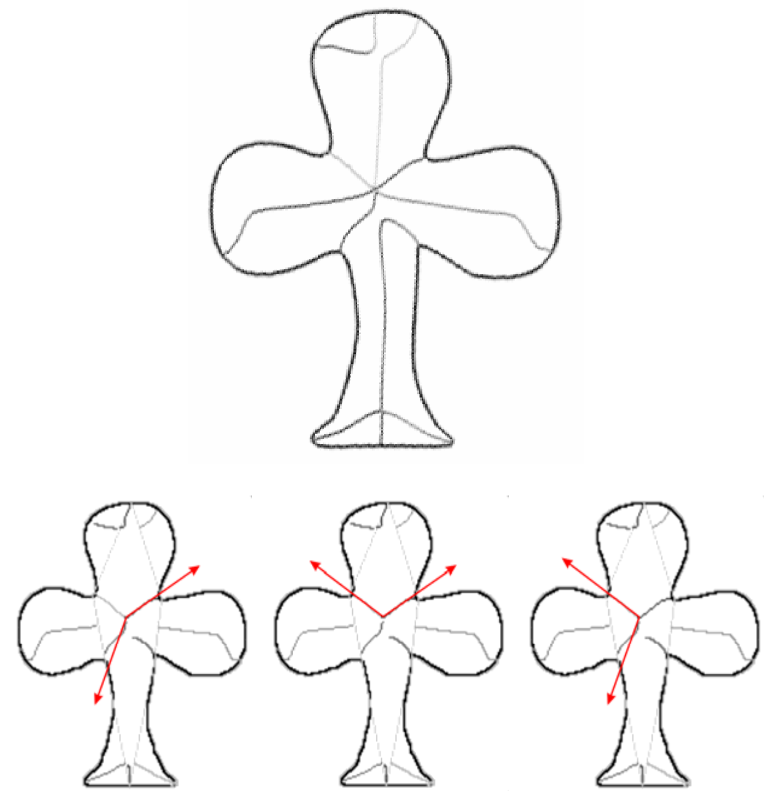


Spatial Organization of Symmetry Branches

- Unique object center
- 1 level of hierarchy



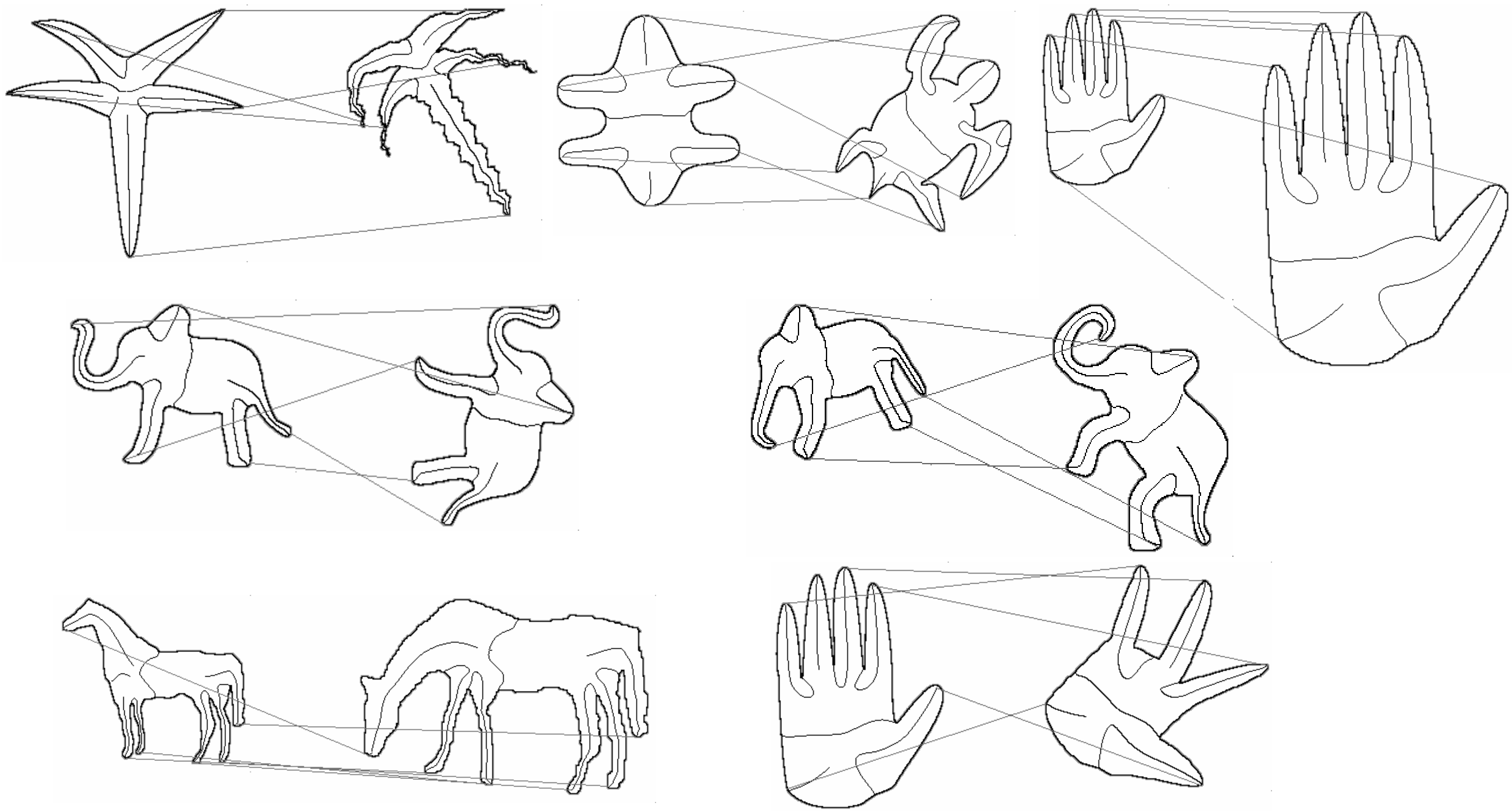




Shape Matching

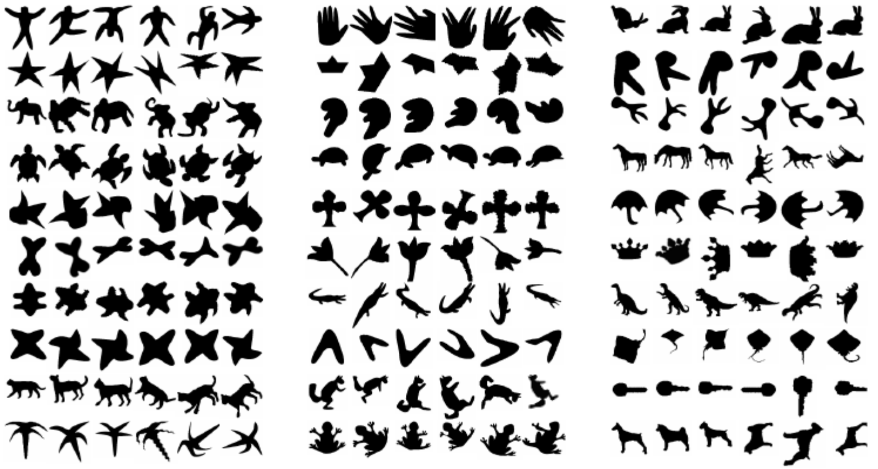
- Branch and Bound
  - Very fast due to small number of skeletal branches
- String-Edit
  (morphing one skeletal string into another)

Some Matching Examples

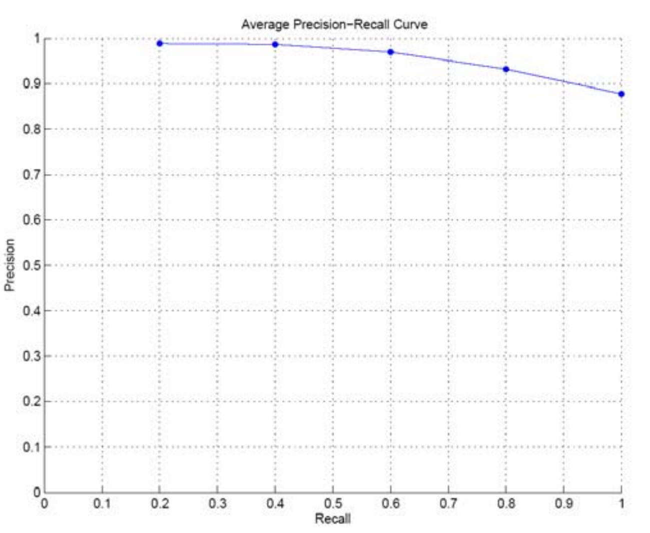


Retrieval Results

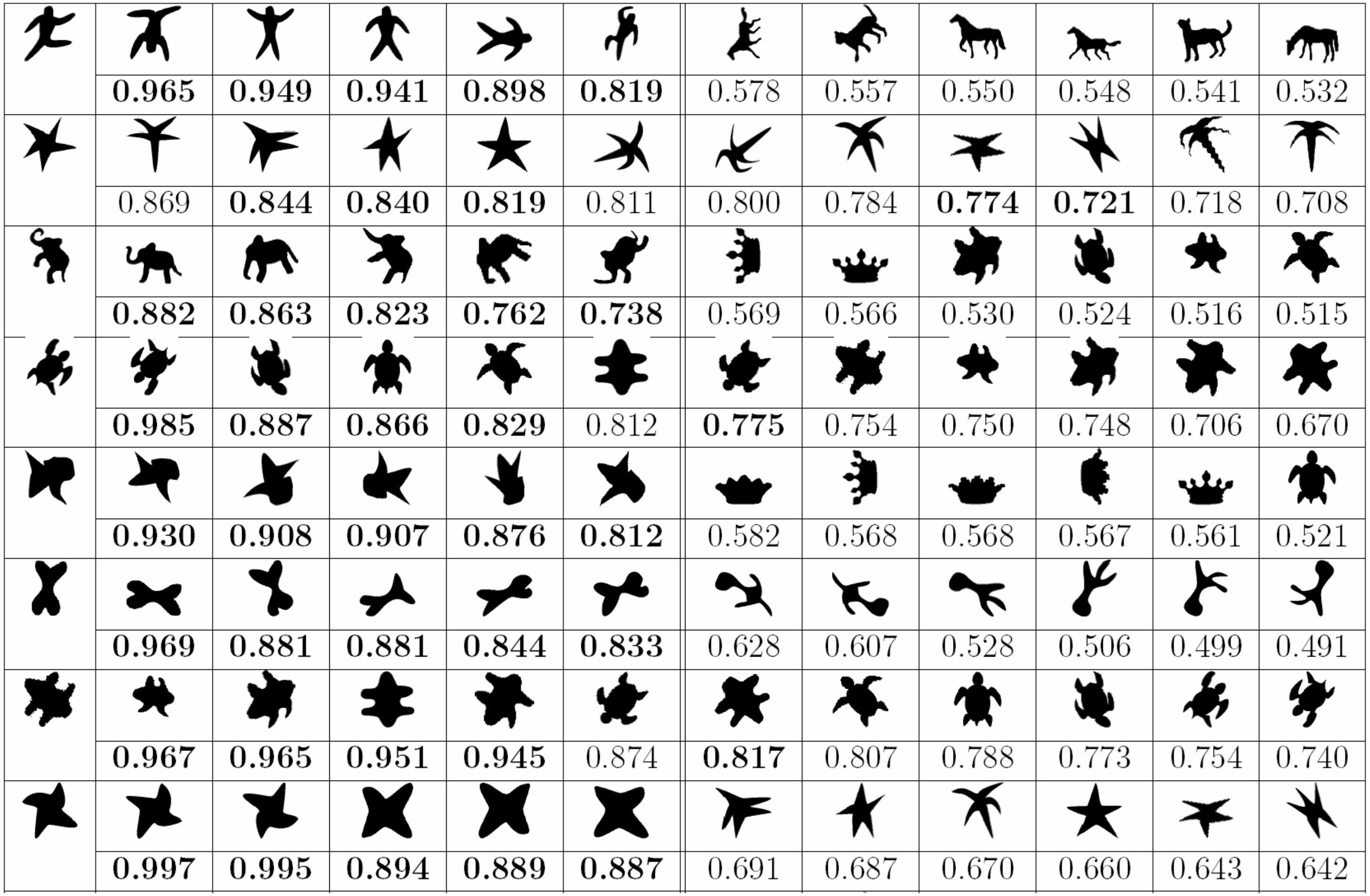
- 180 shapes with 30 categories



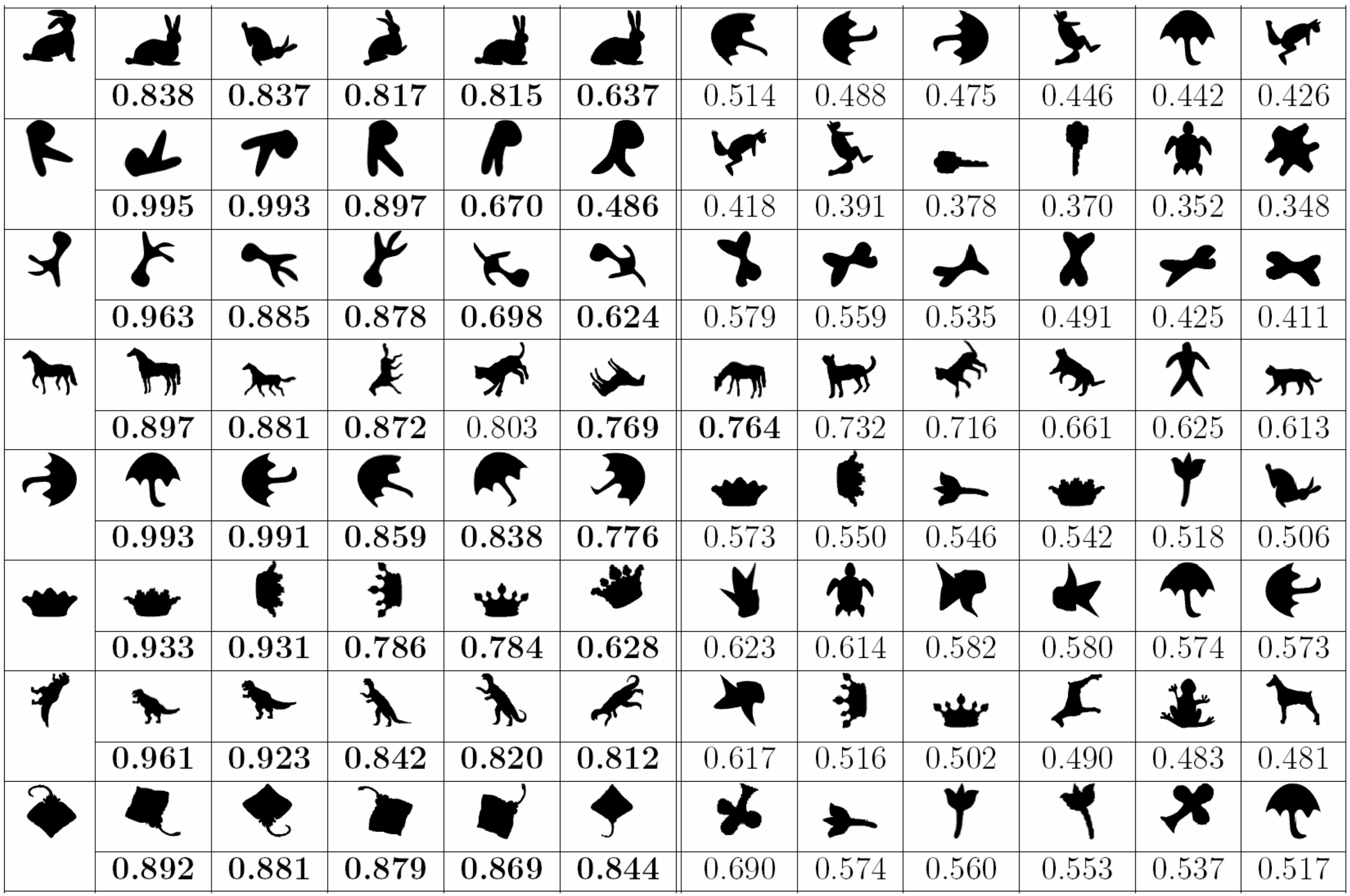
- 91.2% correct retrieval rate in top 6 matches
- Precision is around 0.88 when recall is 1



Retrieval Results (1)

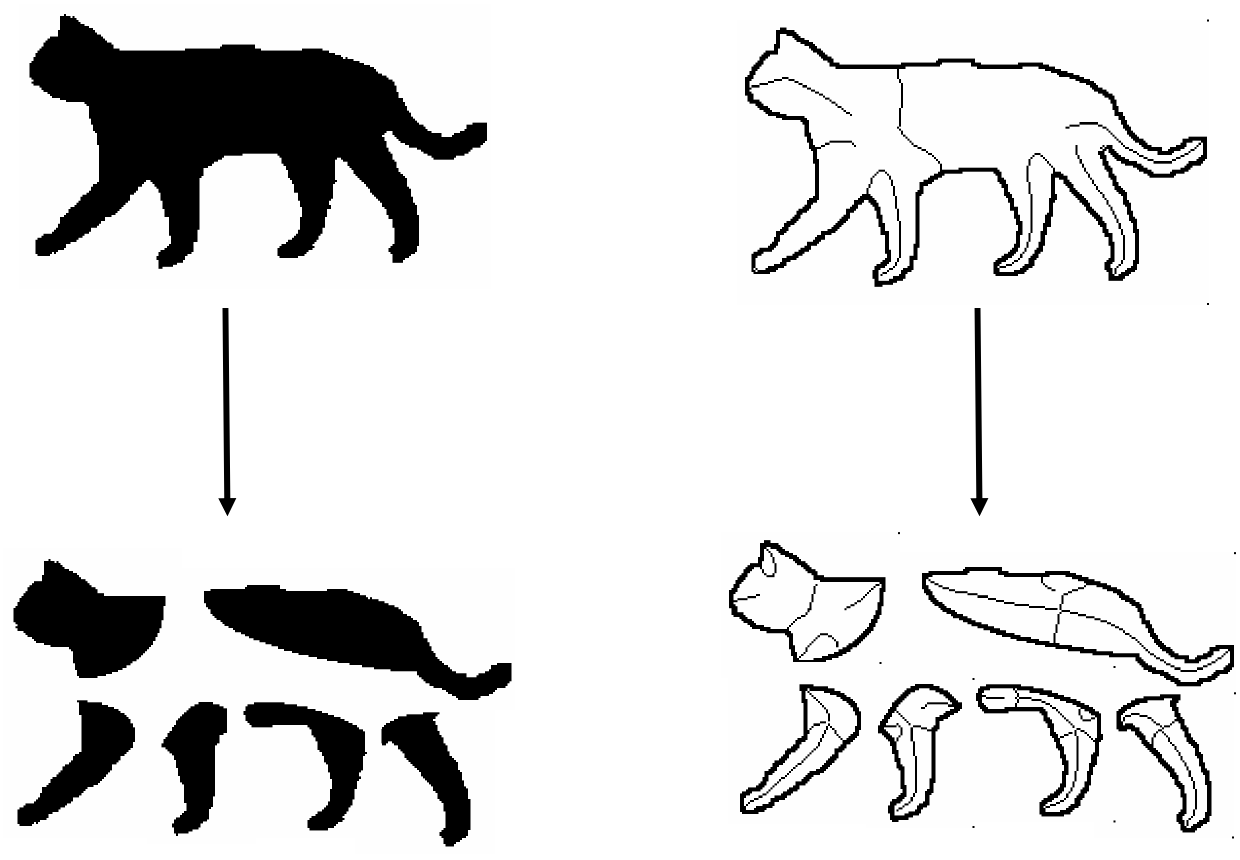


Retrieval Results (2)



Extending 1-level of hierarchy

- Coarse to fine representation

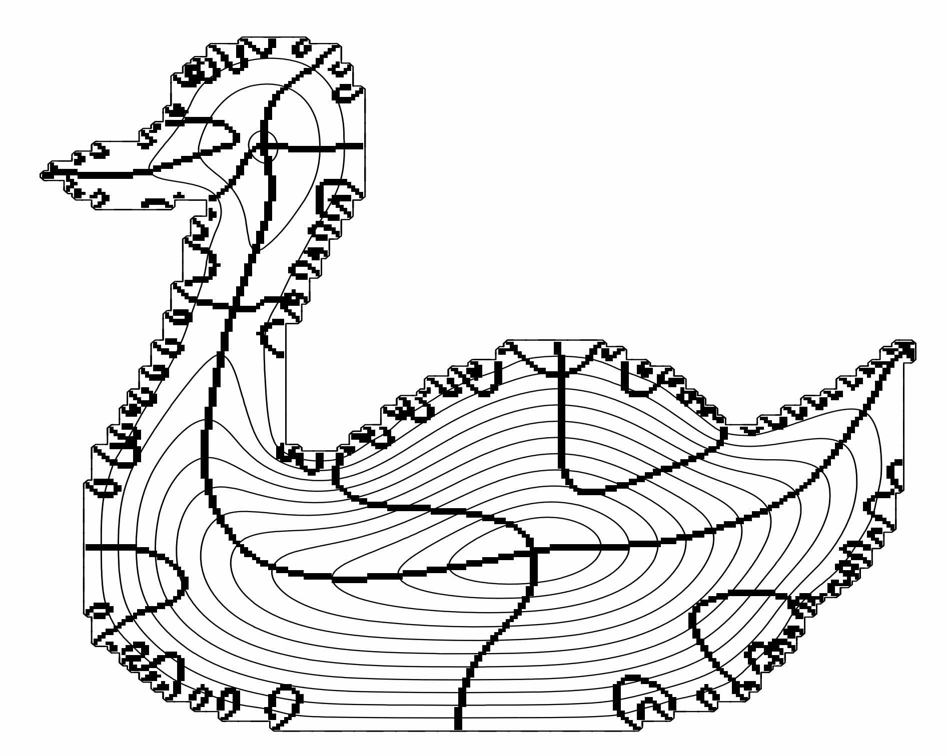


Additional Slides

References

- S. Tari, J. Shah, and H. Pien. A computationally efficient shape analysis via level sets. *MMBIA*, pages 234–243, 1996.
- S. Tari, J. Shah, and H. Pien. Extraction of shape skeletons from grayscale images. *CVIU*, 66(2):133–146, 1997.
- S. Tari and J. Shah. Local symmetries of shapes in arbitrary dimension. In *ICCV*, pages 1123–1128, 1998.
- C. Aslan and S. Tari. An axis-based representation for recognition. In *ICCV*, pages 1339–1346, 2005.

Symmetry Point Computation

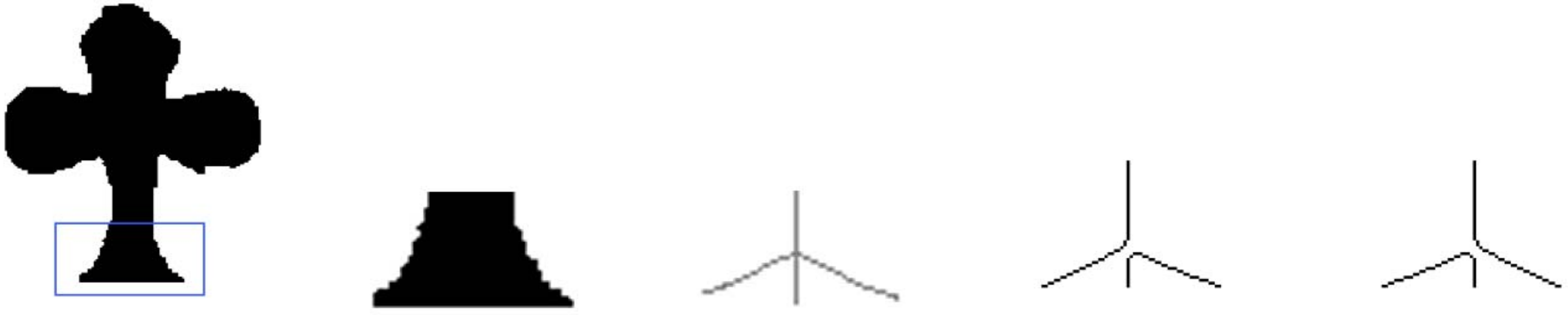


$$\frac{d|\nabla v|}{ds} = 0 \begin{cases} \dfrac{d^2|\nabla v|}{ds^2} < 0 \\ \dfrac{d^2|\nabla v|}{ds^2} = 0 \\ \dfrac{d^2|\nabla v|}{ds^2} > 0 \end{cases}$$

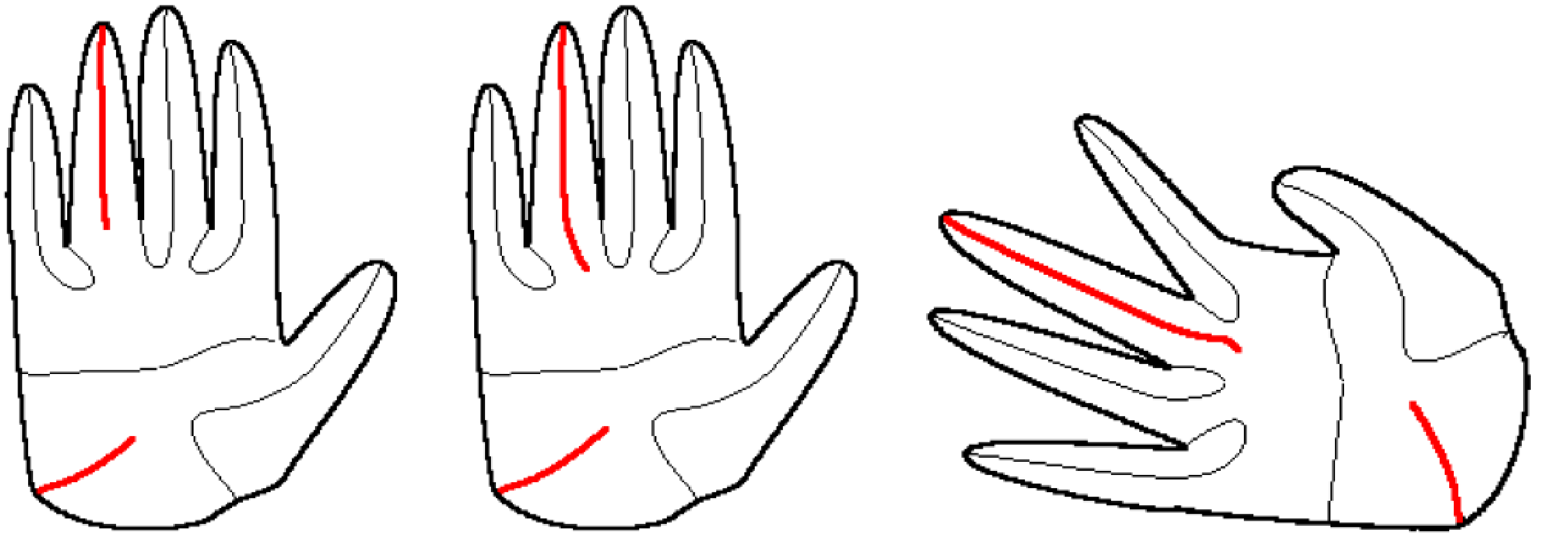
$$\frac{d^2|\nabla v|}{ds^2} = v_{\eta\xi\xi} + \frac{v_{\xi\xi}\left(v_{\xi\xi} - v_{\eta\eta}\right)}{|\nabla v|^2}$$

Handling Triple Junctions

- Breaks down 1-level of hierarchy
- Multiple descriptions



Pruning Major Positive Branches



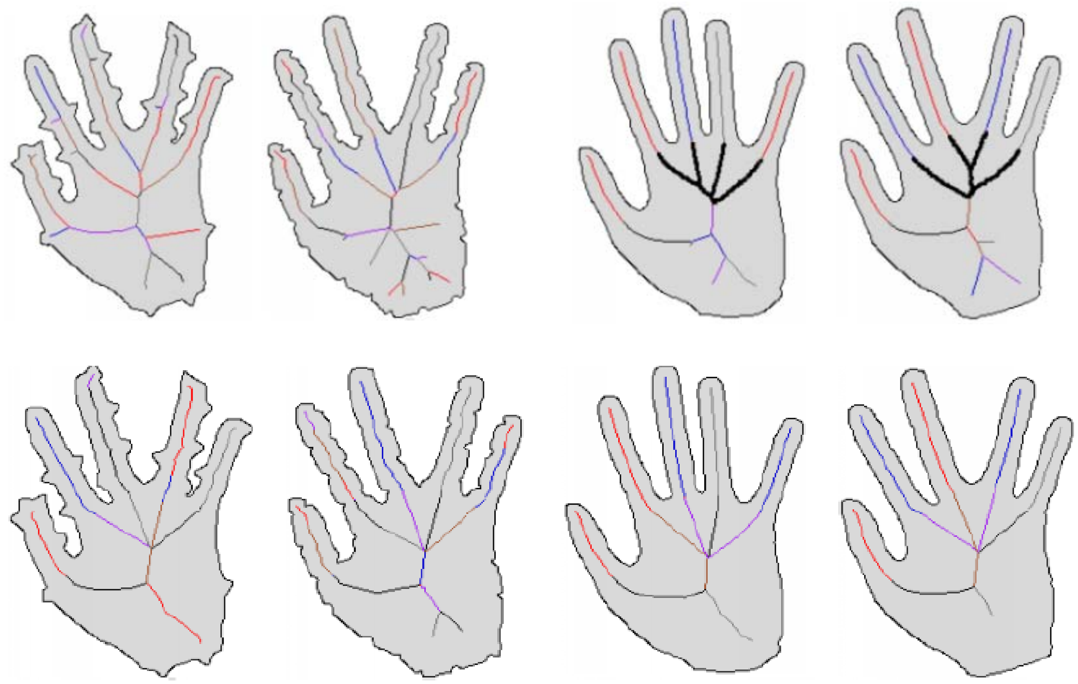
Additional Slides(2)

"*Shapes, shocks, and deformations*" Kimia, Tannenbaum, and Zucker, IJCV, 1995.

$$\frac{\partial C}{\partial t} = \left(\beta_0 + \beta_1 \kappa \right) \vec{N}$$

- Serious computational difficulties in the implementation.
  - Highly nonlinear,
  - Requires a shock-capturing scheme,
  - When diffusion is introduced, first-order shocks (local curvature maxima of the evolving curve) becomes difficult to detect.
    - Large amount of diffusion is NOT possible

Stability vs. Sensitivity



"Canonical Skeletons for Shape Matching",
M. van Eede, D. Macrini, A. Telea, C. Sminchisescu and S. Dickinson,
ICPR, 2006